

# Considerations in mass calibration of pressure balance weights

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## Abstract

A review of mass calibration of pressure balance weights was developed to explain in detail the considerations around the measurements of mass value determination of each weight applied on the pressure balance that use an accurate mathematical model by industrial and accredited laboratories.

The mass value determination of each weight applied on the pressure balance is an important uncertainty contribution in the estimation of the uncertainty measurement of a pressure balance when used for the calibration of another measuring instrument and in the case where the total measurement uncertainty has to be of high level.

There are different considerations have to be applied to take into account: calibration method of mass values, weights densities values, conventional mass, and estimation of the uncertainty measurement of mass value by those that use a pressure balance that use an accurate mathematical model to improve the pressure measurement the best of possible.

*Keywords:* mass, density, calibration, piston, dead weight, pressure balance.

## 1. Introduction

The considerations in the mass calibration of the pressure balance weights described in this work apply on the pressure balance that use an accurate mathematical model by industrial and accredited laboratories.

The knowledge of the required uncertainty on the pressure balance is convenient to choose the adequate procedure and the considerations that must be known.

The mass calibration laboratories used the substitution method (often called The Borda Method). The test weight should be calibrated by comparison against one or more reference weights. In each comparison, the nominal mass of the test weight and the reference weight should be equal. The reference weight should generally be of a higher class of accuracy than the weight to be calibrated.

## 2. Formula of pressure balance

The principle of measurement of a pressure balance (often called pressure balance dead weight) is based upon of balancing the force produced by the measured pressure on a known area with the gravitational force of known loaded weights, as realized with a piston-cylinder assembly. The pressure mathematical model is indicated in equation (1).

$$P = \frac{F}{A} \quad (1)$$

Where,  
N/m<sup>2</sup>,

$P$  is the pressure, and has SI units of pascals 1 Pa = 1

$F$  is the force, N,  
 $A$  is the area, m<sup>2</sup>.



**Figure 1.** Pressure Balance of High Accuracy.

There is a complete formula (that use an accurate mathematical model) where the scope of this method is to determine the masses of the piston with weight carrier and of the individual weight of the pressure balance, and determines the effective area of the piston cylinder assembly as indicated in equation (2).

$$P = \frac{\sum_{i=1}^n m_i \left[ 1 - \left( \frac{\rho_a}{\rho_m} \right) \right] g_i + \gamma C}{A_o \left( 1 + \alpha_{pc} \cdot (t_{pc} - t_r) \right) (1 + bP_n)} \quad (2)$$

Where,  $P$  is the corrected pressure, in Pa,

Pa,	$P_n$	is the pressure of the weight stack in standard conditions, in
	$m_i$	is the mass value of the weight, in kg, is the individual mass value of the weights applied on the piston, including all floating elements,
	$g_l$	is the local acceleration due to gravity, in m/s <sup>2</sup> ,
	$g_o$	is the standard acceleration gravity, $g_o = 9,806\ 65\ \text{m/s}^2$ ,
	$\rho_a$	is the local air density, in kg/m <sup>3</sup> ,
	$\rho_m$	is the mass density, in kg/m <sup>3</sup> ,
	$\alpha_{pc}$	is the linear thermal expansion coefficient of the materials of piston-cylinder assembly, in °C <sup>-1</sup> ,
	$t_{pc}$	is the temperature of the piston-cylinder assembly, in °C <sup>-1</sup> ,
	$t_r$	is the reference temperature of the piston-cylinder assembly, in °C <sup>-1</sup> ,
	$C$	is the circumference of the piston,
	$\gamma$	is the surface tension of the oil,
	$A_o$	is the effective area of the piston-cylinder assembly at a reference temperature $t_r$ (usually 20 °C),
	$b$	is the pressure deformation coefficient of the piston-cylinder assembly.

The mathematical model as indicated in equation (2) is mainly used by the pressure balance of accuracy class 0,005 %, 0,01 % or 0,02 % (High Accuracy), of the value of the test pressure.

There is another formula where the scope of this method is to determine the bias error and the repeatability of the calibrated pressure balance. This is done by determining the pressure generated by the piston-cylinder assembly with weight carrier and of the individual weight of the pressure balance under standard acceleration due to gravity, as indicated in equation (3).

$$P = \sum P_n \cdot \frac{g_l}{g_o} \cdot \frac{1 - \rho_a / \rho_m}{1 - \rho_{an} / \rho_m} \cdot \frac{1}{1 + \alpha_{pc} \cdot (t_{pc} - t_r)} \quad (3)$$

Where,  $\rho_{an}$  is the standard air density,  $\rho_{an} = 1,2\ \text{kg/m}^3$ ,

The mathematical model as indicated in equation (3) is mainly used by the pressure balance of accuracy class 0,05 %, 0,1 % or 0,2 % (Low Accuracy), of the value of the test pressure. In this class of pressure balance under calibration is optional that the user convert a commercial industrial pressure balance to one that use an accurate mathematical model to improve the benefits of the uncertainty measurement of a pressure balance. In addition, the determination of effective area of the piston-cylinder assembly, the determination of the mass density, the determination of pressure deformation coefficient of the piston-cylinder assembly, the measurement of the temperature of the piston-cylinder and the determination of the float position of the piston must be known with sufficient accuracy.

### 3. Uncertainty of the mass determination

The relative uncertainty of the mass determination should not usually exceed 20 % of the likely total measurement uncertainty of the pressure balance to be calibrated [5].

**Example 1**, in the case of one pressure balance with pressure range of 12 to 700 kPa absolute or gauge, the pressure expanded uncertainty is  $1 \cdot 10^{-5}$ . The relative uncertainty of the determination of the masses of the piston with weight carrier and of the individual weights should be within  $2 \cdot 10^{-6}$  according to EAL-G26. This relative uncertainty of mass determination will correspond to the weights in class F1 in OIML R 111-1 [7].

**Example 2**, in the case of one pressure balance with pressure range of 700 kPa to 7 MPa absolute or gauge, the pressure expanded uncertainty is  $5 \cdot 10^{-5}$ . The relative uncertainty of the determination of the masses of the piston with weight carrier and of the individual weights should be within  $1 \cdot 10^{-5}$  according to EAL-G26. This relative uncertainty of mass determination will correspond to the weights in class F2 in OIML R 111-1 [7].

**Example 3**, in the case of one pressure balance with pressure range of 700 kPa to 7 MPa absolute or gauge, the pressure expanded uncertainty is  $1 \cdot 10^{-4}$ . The relative uncertainty of the determination of the masses of the piston with weight carrier and of the individual weights should be within  $2 \cdot 10^{-5}$  according to EAL-G26. This relative uncertainty of mass determination will correspond to the weights in class M1 in OIML R 111-1 [7].

Otherwise, the recommendation OIML R 110 [4] determine the accuracy maximum permissible errors class for the adjustment of the mass of the weights. In this manner, the weight class in OIML R 111-1 that correspond to the Accuracy Class of the Pressure Balance as shown in Table (1).

Accuracy Class Of Pressure Balance	EAL-G26		OIML R 110	
	Relative Uncertainty	According OIML Class	Accuracy Maximum Permissible Errors	According OIML Class
$1 \cdot 10^{-5}$	$2 \cdot 10^{-6}$	E2	$0,1 \cdot 10^{-5}$	E2
$5 \cdot 10^{-5}$	$1 \cdot 10^{-5}$	F1	$0,5 \cdot 10^{-5}$	F1
$1 \cdot 10^{-4}$	$2 \cdot 10^{-5}$	F2	$1,5 \cdot 10^{-5}$	F2
$2 \cdot 10^{-4}$	$4 \cdot 10^{-5}$	F2	$1,5 \cdot 10^{-5}$	F2
$5 \cdot 10^{-4}$	$1 \cdot 10^{-4}$	M1	$5 \cdot 10^{-5}$	M1
$1 \cdot 10^{-3}$	$2 \cdot 10^{-4}$	M2	$16 \cdot 10^{-5}$	M2
$2 \cdot 10^{-3}$	$4 \cdot 10^{-4}$	M2	$16 \cdot 10^{-5}$	M2

**Table 1.** Weights class in OIML R 111-1 according to the Accuracy Class of the Pressure Balance.

#### 4. Materials and density

The calculation of the air buoyancy correction involves the determination of the mass density. Prior to mass determination, the density of the piston, weight carrier and individual weight must be known with sufficient accuracy.

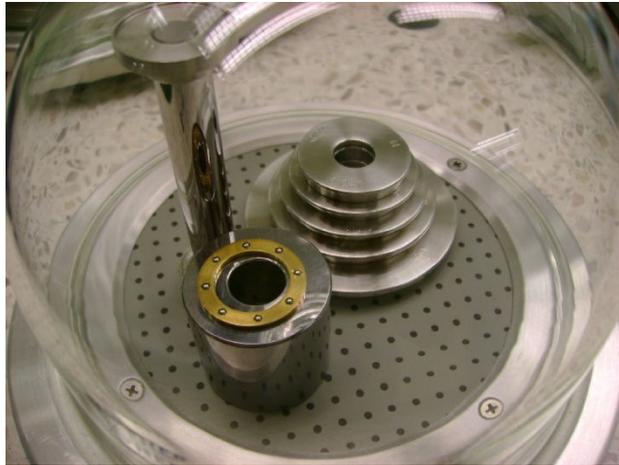
They are made from different materials. If the density are not known, but the material is known, the appropriate assumed density from Table (2) and (3) should be used.

Material	Density	Manufacturer
Stainless Steel	$7\ 895\ \text{kg/m}^3$	Ametek, Pressurements
Brass	$8\ 390\ \text{kg/m}^3$	Chandler, Dewit
Kirkstite	$6\ 700\ \text{kg/m}^3$	Ametek, Ashcroft-Dresser, Terris.
Aluminum	$2\ 710\ \text{kg/m}^3$	Ametek

**Table 2.** List of alloys most commonly used for weights of Pressure Balance Manufacturers.

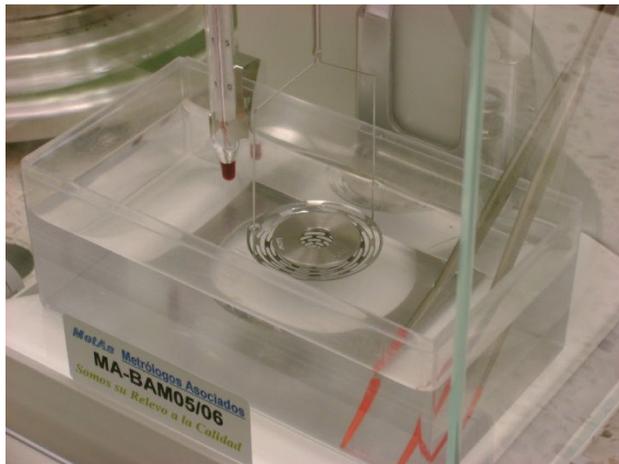
Material	Density	Manufacturer
Tungsten Carbide	$13\ 300\ \text{kg/m}^3$	DH Instruments, DH Budenberg, Ruska, Pressurements
Stainless Steel	$7\ 800\ \text{kg/m}^3$	Ametek, Ashcroft-Dresser, Terris, Chandler, Dewit

**Table 3.** List of alloys most commonly used for Piston of Pressure Balance Manufacturers.

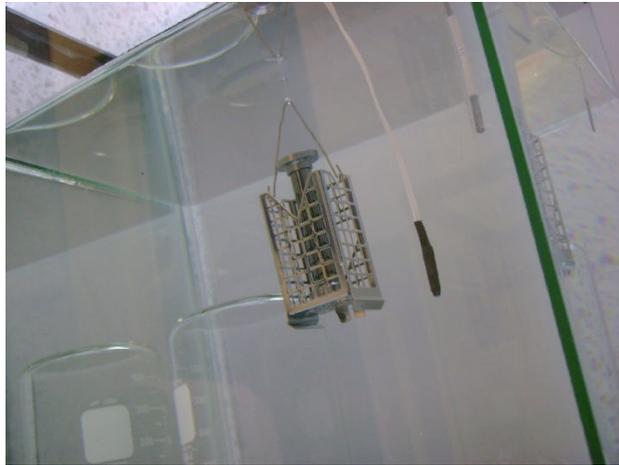


**Figure 2.** Piston and Weights of the Pressure Balance.

If more accuracy is needed, it is necessary to determine the mass density by calibration. This determination can be performed: (1) comparison between test and reference weight in air and comparison between test weight in liquid and a second reference weight in air, (2) volume determination by weighing of the displaced liquid.



**Figure 3.** Density Determination by Hydrostatic Comparison of a Weight (10 g).



**Figure 4.** Density Determination by Hydrostatic Comparison of a Piston (almost 100 g).

## 5. Mass calibration method

The calibration of the masses of the pressure balance shall be determined by a laboratory accredited for such mass measurements.

In general, the “Mass Value” of the standards means its True mass values. The True mass is used to calculate the measured pressure in the pressure balances. The calibrations of dead weights are at a lower accuracy the calibrations of mass standards.

A comparison to a known mass is applied; this mass may consist of one or more reference standards. The reference standards shall be calibrated with traceability to the international kilogram at the Bureau International des Poids et Mésures (BIPM) in Paris.



**Figure 5.** Mass Calibration of a Piston (almost 100 g).

### 5.1 Conventional Mass Calibration (substitution method)

In the conventional mass determination, of the test weight, with the substitution comparison method is made by the equation:

$$m_m = m_R - (\rho_a - 1,2) \cdot (V_R - V_m) + \overline{\Delta m} \quad (4)$$

Where,

- $m_m$  is the conventional mass of the test weight,
- $m_R$  is the conventional mass of the reference weight,
- $\rho_a$  is the air density,
- $V_R$  is the volume of the reference weight,
- $V_m$  is the volume of the test weight,
- $\overline{\Delta m}$  is the the average weighing difference observed between the test and reference weights.

The mass difference between test and reference weights is determined by the follow equations:

$$\overline{\Delta m} = \overline{\Delta L} \cdot \overline{Sb} \quad (5)$$

$$\overline{\Delta L}_i = \left( \frac{(L_2 - L_1) + (L_3 - L_4)}{2} \right) \quad (6)$$

$$\overline{Sb}_i = \left( \frac{m_{ps}}{L_3 - L_2} \right) \quad (7)$$

Where,

- $\overline{\Delta L}$  is the indication difference of the balance between the test and reference weights,
- $\overline{Sb}$  is the inverse sensitivity of the balance,
- $m_{ps}$  is the sensitivity weight,
- $L_{1,2,3 \text{ y } 4}$  is the indications of standard balance,
- $L_1$  is the indication for reference weight,
- $L_2$  is the indication for test weight,
- $L_3$  is the indication for test weight, +  $m_{ps}$ ,
- $L_4$  is the indication for reference weight +  $m_{ps}$ .

## 6. Relation between true mass and conventional mass

The calibration method for mass determination is used mainly in conventional mass. The relation between true mass and conventional mass of an object must be as indicated in equation (8).

$$m_t = m_c + (V_{mt} - V_{8000}) \cdot 1,2 \quad (8)$$

Where,

- $m_c$  is the conventional mass of the object, in kg
- $m_t$  is the true mass of the object, in kg,
- $V_{mt}$  is the volume of the object, in  $m^3$ ,
- $V_{8000}$  is the volume of the object at standard density of  $8\,000\text{ kg}\cdot\text{m}^{-3}$ .



**Figure 6.** Mass Calibration of a Weight (1 000 g).

## 7. Calibration results

The calibration results of the masses of the piston with weight carrier and two individual weight of the pressure balance, are shown in Table (4).

<b>Mass Value (VIM3 5.18)</b>	<b>Volume Value @ 20°C (VIM3 5.18)</b>	<b>Density Value (VIM3 5.18)</b>
<b>g</b>	<b>cm<sup>3</sup></b>	<b>mg/cm<sup>3</sup></b>
98,793 373	8,090 1	12 211,6
118,821 915	38,895 7	3 054,88
1 000,013 24	126,929	7 878,5
9,999 929 8	1,272 7	7 857,2
<b>Mass Instrumental Uncertainty (VIM3 4.23)</b>	<b>Volume Instrumental Uncertainty @ 20°C (VIM3 4.23)</b>	<b>Density Instrumental Uncertainty (VIM3 4.23)</b>
<b>g</b>	<b>cm<sup>3</sup></b>	<b>mg/cm<sup>3</sup></b>
±0,000 044	±0,002 5	±3,8
±0,000 049	±0,002 5	±0,20
±0,000 18	±0,050	±3,1
±0,000 007 3	±0,002 5	±15,4

**Table 4.** Calibration Results.

## Conclusions

The relative uncertainty component of pressure measurements due to uncertainty of mass is numerically equal to the relative value of the mass uncertainty .

The substitution method to the determination of measurement of mass value of each weight can be performance according the level of the total uncertainty of pressure measurement. If we wanted improve the uncertainty of a pressure balance measurement, it is necessary to take the above considerations into account.

The estimation of long-time stability would require to collect the history of mass calibration. A mass calibration method in true mass is totally recommended to improve the method.

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