

Fem analysis for an industrial pressure balance from 3 MPa to 120 MPa in liquid medium

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Abstract:

The main purpose of this work is the numerical and experimental analysis of an industrial pressure balance operating in liquid media up to 120 MPa. The numerical prediction of the pressure distortion coefficient, the piston fall rates, the piston-cylinder elastic distortions is achieved through the Finite Element Method (FEM). Numerical results are compared to experimental values and a sensitivity analysis is performed in order to explain some differences between FEM and experimental results .

Keywords: pressure balance, Finite Element Method, liquid, pressure distortion coefficient.

1. Introduction and pressure balance description

On the basis of a consolidated cooperation between INRiM/University of Cassino and Scandura-FEM, it was decided to test how finite element calculation methods can be a useful tool to predict the performance of a pressure balance designed for industrial use. The interest is because the geometry of the piston-cylinder assembly is, in this case, affected by a radial clearance along the piston-cylinder engagement length larger than one μm and not constant. The pressure balance under investigation is Scandura type MPA-120, 120 MPa full scale in liquid media using a free deformation piston-cylinder unit. The main characteristics of this pressure balance were described in recent papers [1, 2, 3]. The 120 MPa full scale pressure balance is the first of a series of four different piston-cylinder units in tungsten carbide, with different full scales: 120 MPa, 60 MPa, 36 MPa and 12 MPa . Different pressure balance models will be available; from the simplest one, totally mechanical, to the complete one, with the motor to drive piston rotation in a non permanent way and with all the sensors needed to measure and to compensate errors due to the main influence variables.

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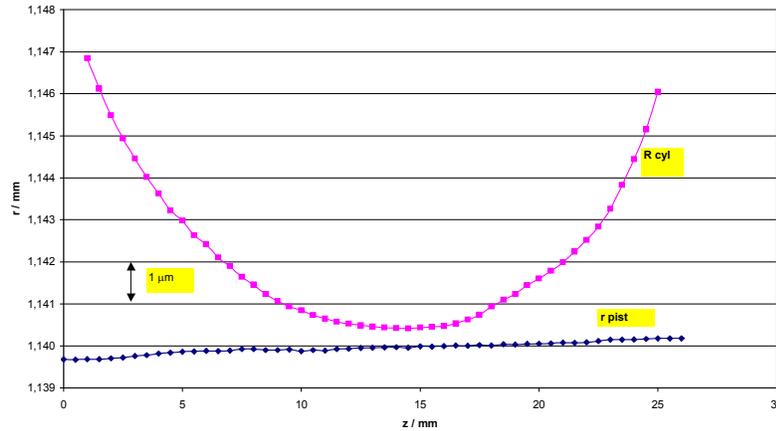


Figure 1 - MPA 120 MPa piston and cylinder radius along the engagement length (from $z = 0$ to $z = 26$ mm).

In Figure 1 the radius of piston and cylinder of the 120 MPa assembly is given, as measured at INRiM with a standard uncertainty of radial measurements of $u(r) = u(R) = 25$ nm, all along the engagement length from 0 value (initial bottom part of engagement length where the maximum pressure is applied) to 26 mm which is the upper part of the engagement length at the upper end of the cylinder. As it can be seen from Figure 1 there is a central zone (from $z = 10$ mm to $z = 18.5$ mm) where the radial clearance is below $1 \mu\text{m}$, while in the remaining part there is an evident hourglass shape which makes the radial clearance varying from about $7 \mu\text{m}$ (close to $z = 0$) to about $6 \mu\text{m}$ (close to $z = 26$ mm). The effective area of piston-cylinder assembly has been determined from pressure cross-floating with one of INRiM pressure national standard. The behaviour of $A_e = f(p)$ points out a good linearity of effective area as a function of pressure, as expected for a free deformation unit. The mean value of effective area of the piston-cylinder unit at atmospheric pressure and at reference temperature of 20°C , is equal to 4.08501 mm^2 and the value of pressure distortion coefficient λ is equal to $2.0 \cdot 10^{-6} \text{ MPa}^{-1}$. The $A_e = f(p)$ is known with an expanded uncertainty of 80 ppm ($1 \text{ ppm} = 10^{-6}$). For this assembly the cross floating sensitivity was always smaller than 12 ppm over the full measuring pressure range. The behaviour of piston fall rate as a function of pressure was experimentally obtained and its maximum value is close to $32 \mu\text{m} \cdot \text{s}^{-1}$ at 100 MPa but affected by a large expanded uncertainty of $7 \mu\text{m} \cdot \text{s}^{-1}$.

2. Basic calculations with fem

All FEM calculations use the same theory and models coupled with iterative methods as described for other pressure balances [4]. Due to large irregularity of the shape, compared with other assemblies, the number of nodes along the clearance was selected to be 261 after a mesh sensitivity analysis done at the maximum pressure of 120 MPa.

For tungsten carbide material a Young modulus $E_p = E_c = (590 \pm 59) \text{ GPa}$ and Poisson coefficient $\nu_p = \nu_c = (0.22 \pm 0.022)$ were used. Fluid is di-ethyl-hexyl-sebacate whose density and dynamic viscosity versus pressure and

temperatures assumes values as given in [4]. The simplified geometry of the model is shown in Figure 2 whereas the corresponding coordinates of the main points are shown in Table 1.

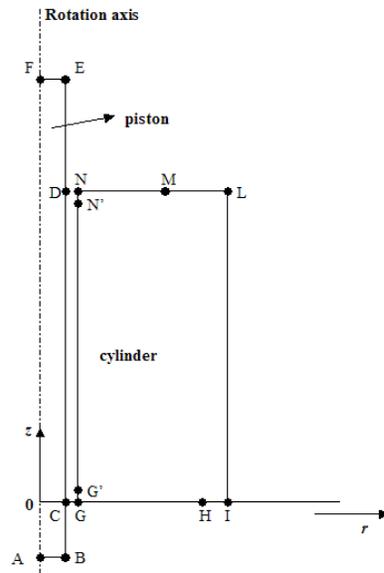


Figure 2 - Baseline model

Table 1 – Coordinates of the main cylinder and piston points

Point	r coordinate mm	z coordinate mm
G	1.148262	0
H	7.5	0
I	8.5	0
L	8.5	26
M	5.0	26
N	1.35	26
N'	1.146045	25
G'	1.146837	1
A	0	-3.5
B	1.14	-3.5
C	1.139678	0
D	1.140183	26
E	1.14	56.5
F	0	56.5

The boundary conditions, in relation to the geometric domain of Fig. 2, are the followings:

Cylinder boundary conditions

- 1) $p(r,z) = p_c(z)$ on $\overline{GG'N'N}$; 2) $p(r,z) = p_a$ on \overline{NM} and \overline{LI} and \overline{IH}
- 3) $p(r,z) = p_m$ on \overline{GH} ; 4) $w(r,z) = 0$ on \overline{HI} ;
- 5) $w(r,z) = 0$ on \overline{LM}

where p_c , p_a and p_m are the pressure along the clearance, the atmospheric pressure and the measured pressure, respectively. In particular, as regards the cylinder boundary condition 5), it was also replaced by a contact pressure of triangular shape, with its maximum equal to the measurement pressure and placed in the middle point of \overline{LM} and environmental pressure p_a on M and L points. This test was made only at the maximum measurement pressure of 120 MPa producing negligible contributions.

Piston boundary conditions

- 1) $p(r,z) = p_c(z)$ on \overline{CD} ; 2) $p(r,z) = p_a$ on \overline{DEF} ;
- 3) $p(r,z) = p_m$ on \overline{AB} and \overline{BC}
- 4) $u(r,z) = 0$ on \overline{AF} ; 5) $w(r,z) = 0$ on \overline{FE}

FEM numerical results for the baseline model and for different measuring pressures p_m values from 3 to 120 MPa can be summarised as follows. Figure 3 gives the normalised pressure distribution p^* in the clearance versus the normalised axial coordinate z^* in the piston-cylinder engagement length. The dimensionless coordinate z^* is referred to the axial engagement length of the piston-cylinder engagement length ($z^* = 0$ and $z^* = 1$ respectively for the initial and final piston-cylinder engagement length). Figure 3 gives as well the values of the radial clearance gap $h / \mu\text{m}$ versus the normalised axial coordinate z^* . From Figure 3 it can be noted that:

- the trend of the dimensionless pressure p^* in the clearance is highly non linear and also not so much dependent on the measured pressure p_m .
- the radial clearance gap has a marked parabolic behaviour similar to the undistorted one

From FEM results it can be obtained, by least squares linear regression, a value of $A_0 = 4.084651 \text{ mm}^2$ and a value of $\lambda = 1.36 \cdot 10^{-6} \text{ MPa}^{-1}$.

The above mentioned A_0 (FEM) value only differs of 2.5 ppm from the value obtained by calculations following Dadson theory that gives as a result the value of 4.084641 mm^2 .

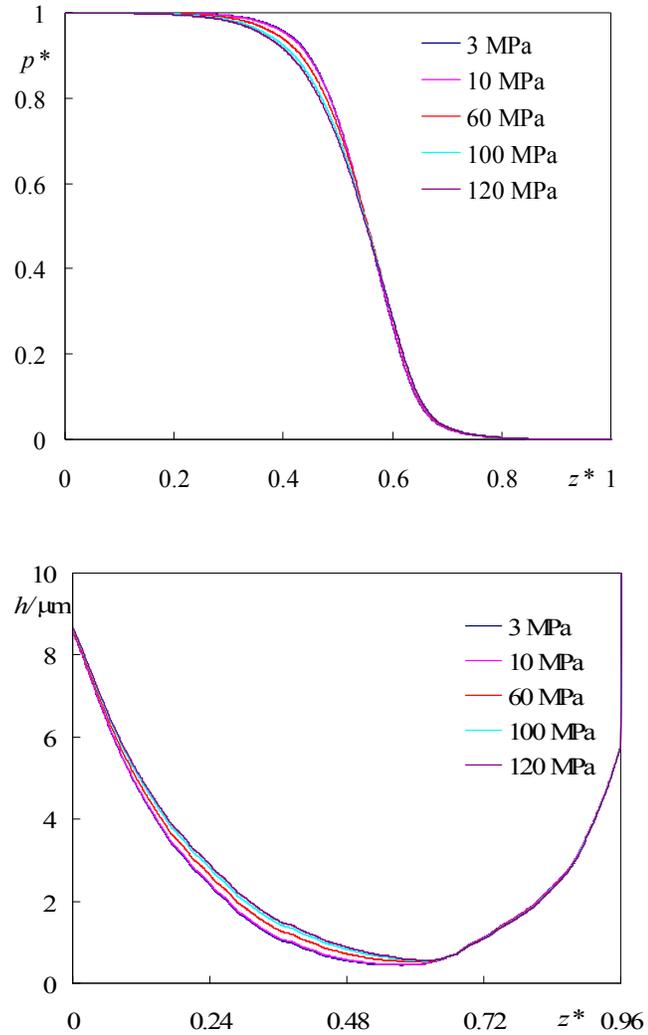


Figure 3 – Normalized pressure distribution p^* in the clearance and radial clearance gap h at different measuring pressures versus the normalized piston-cylinder engagement length z^*

Different sensitivity calculations were made, using the potentiality of FEM methods. For example, at 120 MPa pressure, the sensitivity variations of λ due to an assumed $\pm 10\%$ variations of the elastic constants of the piston and cylinder materials were calculated. The main results are given in the first part of Table 2.

Table 2 - Sensitivity coefficients of the pressure distortion coefficient and sensitivity variations of the piston fall rate due to geometry variations (in both cases at 120 MPa).

quantity	Std. uncert.	Sensitivity coefficient	Std. uncertainty
x_i	$u(x_i) / x_i$	$\partial\lambda/\partial x_i$	$u(\lambda)/\lambda \cdot 10^2$
$E_p = 590 \text{ GPa}$	$10 \cdot 10^{-2}$	$4.01 \cdot 10^{-10} \text{ GPa}^{-2}$	1.7
$E_c = 590 \text{ GPa}$	$10 \cdot 10^{-2}$	$-2.64 \cdot 10^{-9} \text{ GPa}^{-2}$	11.0
$\nu_p = 0.22$	$10 \cdot 10^{-2}$	$1.86 \cdot 10^{-10} \text{ MPa}^{-1}$	3.0
$\nu_c = 0.22$	$10 \cdot 10^{-2}$	$1.46 \cdot 10^{-7} \text{ MPa}^{-1}$	2.3

quantity	Std. uncert.	Std. uncert.	Std. uncert.
x_i	$u(x_i)$	$u(v) / \mu\text{m}\cdot\text{s}^{-1}$	$u(v) / v \cdot 10^2$
Gap width	+ 0.2 μm	15	103
Gap width	- 0.2 μm	- 8.7	- 61
Gap shape conicity	0.2 μm / 26 mm (divergent)	- 1.4	- 9.5
Gap shape conicity	0.2 μm / 26 mm (convergent)	1.2	8

It has to be pointed out, similarly as what was obtained for other pressure balances, that the largest contribution to uncertainty derives from the assumed 10 % standard uncertainty of Young modulus coefficient of the cylinder that will produce a calculated standard uncertainty on λ of 11 %. The sensitivity variations on piston fall rate due to assumed geometry variations were also calculated. The main results are given in the second part of Table 2. As it can be seen an extremely high standard uncertainty of piston fall rate (103 %) is reached just for a variation of + 0.2 μm of the gap width and by the contrary, effects of the gap shape conicity are lower than the ones for example seen in the case of almost constant radial gap. This depends on the fact that the applied conicity variations are negligible compared to the undistorted irregular shape of the cylinder. Variations of λ and piston fall rate were also analysed for an assumed 2.5 % variations of the fluid properties finding negligible influences.

3. Comparison with experimental results

All FEM calculated results are compared with experimental results in terms of effective area and piston fall rates versus measured pressure. In Figure 4 experimental results of $A_e = f(p_m)$ are compared with calculated FEM results and similarly in Figure 5 the comparison is made for the piston fall rate.

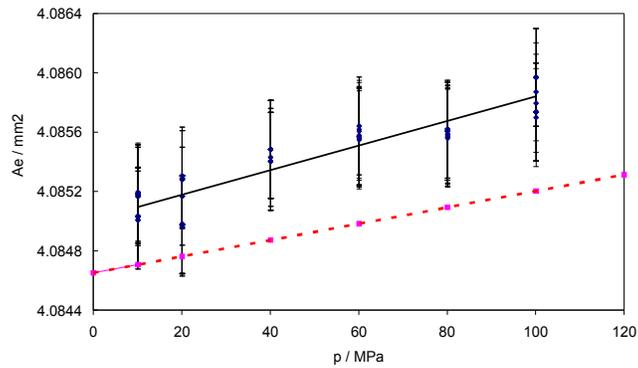


Figure 4 - Experimentally measured $A_e = f(p_m)$ as determined by pressure cross floating compared with FEM results (- - - -).

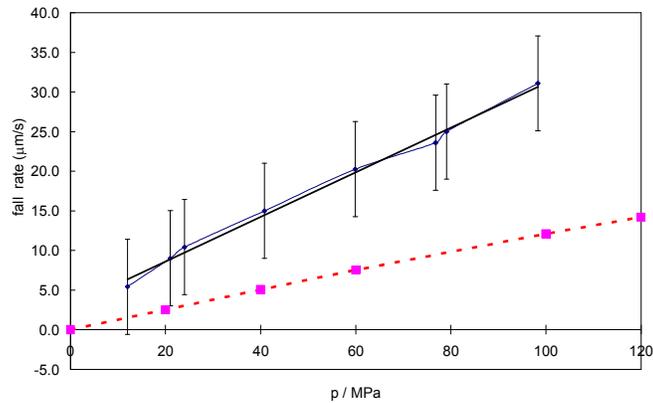


Figure 5 - Experimental piston fall rate versus pressure compared with FEM results (- - - -).

As it can be seen from Figure 4 FEM calculations underestimate the behaviour of effective area versus pressure. The difference in A_0 between experimental value and FEM value is 88.3 ppm, a little outside of 80 ppm expanded uncertainty experimentally evaluated. The difference on the pressure distortion coefficients is of 33 %, also in this case it is a relatively high difference. This difference can only be explained by a larger than expected difference in the cylinder elastic constant (an assumed 10 % variation of E_c contributes to about 11 % difference in λ) while the difference in shape geometry for imposed variations of only $\pm 0.2 \mu\text{m}$ can be as a maximum, responsible of λ variation lower than 2 %.

As it can be seen from Figure 5 FEM calculated values underestimate the piston fall rates and are not in agreement with experimental values that are affected by an estimated expanded uncertainty of $7 \mu\text{m}\cdot\text{s}^{-1}$. A maximum difference of 61 % at 100 MPa is shown. This large difference cannot be explained by underestimation of uncertainty on elastic constants but it is evident that in this case (see Table 2) an underestimation of gap width can be essential for the explanation of differences.

4. Discussions and further calculations

During experimental measurements, in order to better understand the reasons of pressure cross floating sensitivity a little higher than previous tests, different additional measurements have been made (magnetisation effects, electrostatic charges, effect of different position of the piston in respect to cylinder and of different rotation rate) without finding any suspicious effect that could be the reason of foreseen difference. Additional measurements on MPA 120 MPa cylinder gave evidence of a problem of orthogonality between the bottom surface of the cylinder and its vertical axes. This non-perfect orthogonality of surfaces can give rise to an error of verticality of the cylinder that is extremely difficult to be evaluated as the surface of the cylinder is at its bottom side pushed on an O-ring and in its upper side is closed by a counterbalancing nut. Even when the bottom surface of the cylinder was grinded and polished, it is difficult to verify if the error of verticality in respect to the bottom surface is much lower than a few μm . In order to better understand this problem (remember that here the cylinder is dimensionally not constant along the engagement length) more work was done in order to simulate the verticality possible error of the cylinder (because this will affect the dimensional radius of the cylinder along the engagement length) in order to verify the consequences on A_0 , λ and piston fall rate due to an orthogonality error. For this reason it was applied to the cylinder a rotation angle α as given in Figure 6.

Figure 6- Modified model with rotation angle α

In FEM analysis the effects of the variation of the displacement $\Delta X = XX'$ of the point I ($z = 0$, $r = 8.5 \text{ mm}$) both on the effective area A_e and on the piston fall rate v have been investigated. It must be pointed out that a positive displacement, $\Delta X > 0$, results in a counter-clockwise rotation of the cylinder and, consequently, in a reduction of the gap especially near the N' point. An imposed ΔX displacement of $+ 1 \mu\text{m}$ will correspond to a Δr displacement of N' point of $-2.94 \mu\text{m}$.

The effective area and the piston fall rate are in all cases extremely dependent from the rotation of the lower surface of the cylinder. The situation is of extreme importance because of the original dimensional shape of the cylinder. As it can be seen in Figure 7 respectively for A_e and piston fall rate trends versus pressure as a function of different values of ΔX (please note that the piston fall rate scale is of logarithmic type) the effective area and piston fall rate variations are extremely high.

In both cases as ΔX increases the values of effective area and piston fall rates also increase and this has a big effect on A_0 of the order of 1617 ppm/ μm and on λ of the order of 18.5 %/ μm so it is highly possible that this can be an explanation of the differences between experimental values and FEM computed ones.

5. Conclusions

For the first time that an iterative numerical method is applied to a unit having a cylinder with relatively high and not constant radial clearances from less than 1 μm in its central engagement length to 6-7 μm in the upper and lower ends. The main conclusions are the followings:

1. the piston-cylinder unit here presented has an acceptable geometrical quality for piston-cylinders of pressure balances for industrial uses;

2. the quality of the piston-cylinder assembly is confirmed by the regular behavior of rotation speed as a function of time. Starting at approximately 30 revolutions per minute (rpm), after 30 minutes the rotation speed of the assembly is only 25 rpm and the pressure sensitivity, derived from cross-floating against a national standard was found to be better than 12 ppm;

3. the piston fall rate, measured at the pressure of 100 MPa, using as pressurized medium the di-ethyl-hexyl-sebacate is about $32 \mu\text{m}\cdot\text{s}^{-1}$, the differences with FEM numerical data (of the order of 61 % at 100 MPa) have been investigated;

4. the behavior of effective area as a function of pressure is strictly linear as expected for a free deformation piston-cylinder unit. The difference in A_0 values between experimental and FEM calculations (88.3 ppm) and the differences on λ value between experimental and FEM values (33 %) have been investigated ;

5. from FEM calculations considering a possible error of orthogonality, it is possible to derive the following considerations:

- small variations of ΔX give rise to high variations of v and A_0 . For this reason it is highly important that extremely accurate surfaces are made on top and bottom cylinder and that orthogonality of such surfaces in respect to rotational axis of the cylinder is obtained, similarly it is appropriate for cylinder (dimensionally of the type here presented) to have a unique possible mounting position ;

- positive values of ΔX cause reduced differences between experimental and numerical results in terms of v and A_0 ;

- even if the experimental values of v and A_0 are achieved for different values of ΔX ($\Delta X=0.162 \mu\text{m}$ for v and $\Delta X=0.0487 \mu\text{m}$ for A_0 , considering counter-clockwise rotation of the cylinder axis), the displacement values indicate that an orthogonality defect could be one of the reason of the differences founded between FEM and experimental values of v and A_0 ;

- considering the pressure distortion coefficient differences, another possible reason for discrepancy can be due as well to the overestimation of Young modulus of the cylinder (remembering that a 10 % estimated uncertainty on E_c will give rise to a $u(\lambda)/\lambda$ of 11 %).

The relative expanded uncertainty ($k = 2$) of pressure measurements for the here described pressure balance equipped with MPA 120 MPa type - assembly in the measurement pressure range from 3 MPa to 120 MPa, is always between 100 and 80 ppm of the applied pressure.

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