3D IMAGE FORMATION IN TRANSMITTED PARTIALLY COHERENT AND INCOHERENT LIGHT APPLIED TO DIMENSIONAL INSPECTION

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Abstract:

The peculiarities of 3D objects image formation with clear shadow projection under their illumination by partially coherent and perfectly incoherent light based on the constructive theory of 3D objects formation are investigated. Threshold algorithms for determining the position of 3D object geometric boundary, taking into account its thickness, the light source angular size and the projection system angular aperture are developed. These algorithms are based on the application of true (calculated) threshold or standard one using the corrective component for threshold. The cases of weak and strong 3D object volumetricity for partially coherent and incoherent illumination are studied. The analytical equations for these algorithms are given. It has been shown that their use can significantly improve the measurement accuracy of extended objects.

Keywords: 3D Diffraction, Shadow Method, Dimensional Inspection, Partially Coherent Light

1. INTRODUCTION

The shadow systems due to their high precision and operation speed as well as broad measurement range are widely used among optical means of noncontact dimensional inspection in industry [1]. The essence of the measurement shadow method consists in finding of the position of object's true boundary. It allows us to determine its different geometrical dimensions, including width, length, holes diameter, etc. In case of flat objects (zero thickness) the true boundary is determined using shadow image thresholding, either at 25% of light intensity illuminating the object (coherent illumination) or at 50% of intensity (incoherent illumination) [2]. Under industrial inspection of real 3D (thick) objects due to the volumetricity takes place the boundary shift position [3, 4], which is proportional to Fresnel zone. This leads to the need to adjust the threshold level when finding the true position of 3D object boundary.

Due to known problems occurring under the use of coherent illumination (first of all, speckle noises [5]) the more perspective is the quasi-monochromatic partially space coherent illumination for inspected 3D objects using, for instance, LED. Such illumination was effectively used by us for measurement system using Fresnel images of the inspected objects [6].

As shown in [7] the structure of 3D object shadow image depends on some parameters, including system angular aperture, object's thickness, light source angular size.

The peculiarities of 3D object images formation in a diffraction-limited system under their illumination by partially coherent and perfectly incoherent light are studied. The mathematical methods for calculation of intensity distribution in photodetector plane for partially coherent light are presented. The methods for analytical determination of 3D object image shadow boundary shift and its correction depending on the object's thickness and the optical system parameters are proposed. These algorithms allow us to increase considerably the measurement precision of 3D objects by the shadow method.

2. THE SHADOW MEASUREMENTS FOR 3D OBJECTS IN PARTIALLY COHERENT LIGHT

The optical scheme of the shadow system for inspection of 3D objects is presented in Fig. 1. Extended quazi-monochromatic source 1 with angular sizes $2\theta_S$ illuminates a thick edge with width d through objective of lens 2. Projection lenses 4 and 6 form object shadow image on photodetector matrix 7. Aperture diaphragm 5 with angular sizes $2\theta_{ap}$ is located at focal distance from the lens 4.

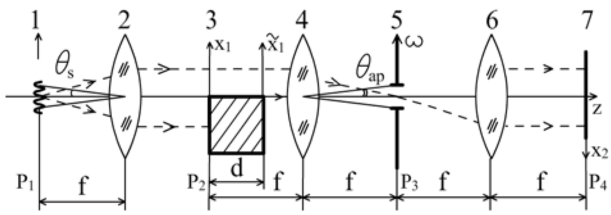

Fig. 1: Optical scheme for formation of 3D object's image in a diffraction-limited system

The image profile of the 3D object's edge can be seen in Fig. 2.

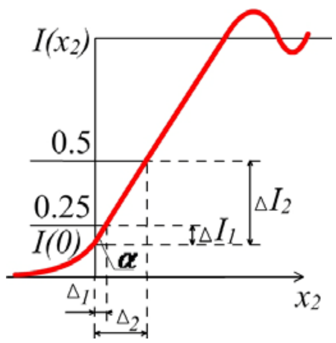

Fig. 2: Image profile of the 3D object's edge on photodetector matrix. Normalized intensity $I(x)$ vs. coordinate x_2

The value of threshold $I_{thr} = I(0)$ depends on some parameters, including angular size of the light source and the aperture diaphragm, as well as the object's volumetricity. For flat objects ($d = 0$) illuminated by coherent light the threshold is equal to $I_{thr}^{(1)} = 0.25$ ($\theta_s \to 0$), and in case of incoherent light illumination the threshold equals $I_{thr}^{(2)} = 0.5$ ($\theta_s \to \pi/2$). Under partially coherent illumination the threshold value is intermediate within the limits of $0.25 < I_{thr} < 0.5$ ($0 < \theta_s < \pi/2$) [2].

The influence of 3D object's thickness on its image profile is determined by the ratio of critical diffraction angle $\theta_{cr} = \sqrt{\lambda/d}$ (under which the volume effects become significant) to the angular aperture size $2\theta_{ap}$ [3, 4] (λ is wavelength). If at $\theta_{cr} >> \theta_{ap}$ the weak volume effects occur, so at $\theta_{cr} << \theta_{ap}$ their effects are sufficiently significant.

3. ALGORITHMS FOR DETERMINATION OF 3D OBJECT'S BOUNDARIES IN PARTIALLY COHERENT LIGHT

Two algorithms to determine the position of the geometric boundary of thick edge, which plane perfectly absorption surface coincides with optical axis ($x_1 = 0$), have been developed. The first one is based on the use of threshold $I_{thr} = I(0)$ (Fig. 1) that takes into account the angular source size $2\theta_s$ and object's thickness d. For calculation we used the constructive theory of image formation for the thick objects with the sharp shadow projections [3]. One has shown that the normalized light intensity in the thick edge image in point $x_2 = 0$ (that coincides with boundary geometric position) under $\theta_s << \theta_{ap}$ is equal to:

$$I_{thr} = I(0) = 0.25 + \frac{1}{3\pi^2}\frac{\theta_s^2}{\theta_{ap}^2} - \frac{\theta_{ap}}{\sqrt{2}\pi\theta_{cr}}. \qquad (1)$$

It is seen that value $I(0)$ is determined by three components. The first term of the equation corresponds to the case when a flat object is illuminated with point axial light source (coherent illumination). The second one refers to the influence of the final angular size source and results in increasing of the threshold level. At last the third component is determined by the object volumetricity which decreases the threshold level $I(0)$. Thus, at given parameters of d and θ_{ap} one has an opportunity, by choosing the angular source size $2\theta_s$, to compensate the effect of the object's volumetricity on the change of light intensity at the point of geometric position of the thick edge boundary ($x_2 = 0$). It is possible due to different signs of the second and third terms in Eq. (1).

Computer modeling has allowed obtaining the dependency of the threshold level on the reduced (to aperture θ_{ap}) light source size (θ_s / θ_{ap}) for the flat edges (Fig. 3). Under $\theta_s = \theta_{ap}$ threshold level jump is observed. When $\theta_s / \theta_{ap} < 0.7$ and $\theta_s / \theta_{ap} > 1.5$ coincidence is observed with an error of less than 2%. The theoretical results as well as computer modeling ones are in good agreement.

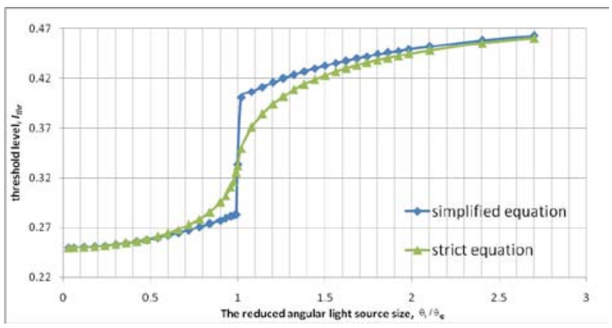

Fig. 3: The threshold level I_{thr} for shadow image vs. the reduced angular light source size θ_s / θ_{ap} for the flat edges

The second developed algorithm envisages the determination of the position of the 3D edge's boundary using the two standard thresholds: $I_{thr}^{(1)} = 0.25$ and $I_{thr}^{(2)} = 0.5$. It is important that high precise determination of the boundaries requires introduction of two corrections $\Delta 1$ and Δ_2 that are determined by the following equations (Fig. 2):

$$\Delta_1 = \frac{0.25 - I(0)}{I'(0)} = \frac{\Delta I_1}{tg\alpha}, \qquad \Delta_2 = \frac{0.5 - I(0)}{I'(0)} = \frac{\Delta I_2}{tg\alpha}, \qquad (2)$$

where $tg\alpha = I'(0)$ is the slope angle of the 3D edge's image profile under the selected level. In this case one has to know the value of $I'(0)$. For finding this value it is necessary to determine a derivative of the output intensity $I(x)$ (Fig. 2) in a point $x = 0$. It's established that at $\theta_s << \theta_{ap}$:

$$I'(0) = \frac{2\theta_{ap}}{\lambda} + \frac{4}{\sqrt{2}\pi\lambda\theta_{cr}}\left(\frac{1}{3}\theta_s^2 - \theta_{ap}^2\right) \qquad (3)$$

Taking into account Eq. (2) and Eq. (3), for corrections Δ1 and Δ2 one can obtain the following:

$$\Delta_1 = \frac{\lambda}{\theta_{ap}}\left[\frac{3}{2\sqrt{2}\pi}\frac{\theta_{ap}}{\theta_{cr}} - \frac{1}{3\pi^2}\frac{\theta_s^2}{\theta_{ap}^2}\right]$$

$$\Delta_2 = \frac{\lambda}{\theta_{ap}}\left[0.25 + \frac{3}{2\sqrt{2}\pi}\frac{\theta_{ap}}{\theta_{cr}} - \frac{1}{3\pi^2}\frac{\theta_s^2}{\theta_{ap}^2}\right] \qquad (4)$$

According to Eq. (4), the values of the corrections depend on the angular aperture size, the angular light source size and the critical angle. The corrections can be minimized through choice of the parameters system due to the different signs of the components in Eq. (4).

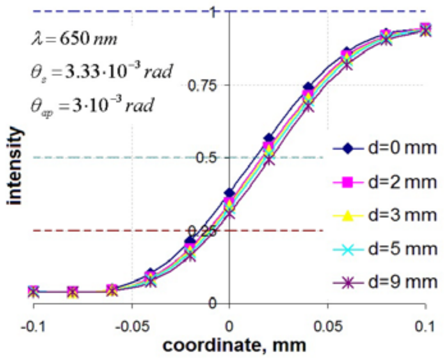

Fig. 4: The calculated 3D edge image intensity profile $I(x_2)$ vs. coordinate x_2 (plane P_4 in Fig. 1) at various object thickness d

Figure 4 shows the calculated volumetric edge image profiles with variable thickness d within 0 ÷ 9 mm. We emphasize that the angular size of the source in the calculations was slightly more than the angular size of the aperture of the system. It can be seen that when d is varied the shift of profile relative to the position of the object real edge takes place (image profile structure is similar).

In this case, the tilt angle α of edge profile of 3D images is varied slightly. The main contribution to the angle α gives the first diffraction term $2\theta_{ap}/\lambda$. The influence of the angular dimensions of the source on angle α (second term in Eq. (3)) in the above ratios of θ_s , θ_{ap} and θ_{cr} may be negligible.

As already mentioned above an important parameter of the system affecting the image formation is the reduced angular light source size θ_s / θ_{ap} . Depending on the ratio of these values, the behavior of the threshold level is varied.

We have studied the case of large $\theta_s : \theta_{ap} << \theta_s << \theta_{cr}$. As result of calculations the following equation was obtained:

$$I(0) = I_{thr} = 0.5 - \frac{1}{\pi^2}\frac{\theta_s}{\theta_{ap}} - \frac{\theta_{ap}}{\sqrt{2\pi}\theta_{cr}} \qquad (5)$$

Borderline situation $\theta_s = \theta_{ap}$ is of particular interest. In this case the threshold level according to our calculations is equal to I_{thr} = 0.333 regardless of the angular sizes of the source and the aperture. It can be used under the development and producing the systems for dimensional inspection.

4. ALGORITHMS FOR DETERMINATION OF 3D OBJECTS IN PERFECTLY INCOHERENT LIGHT

In practice using incoherent light is more preferable than coherent or partially coherent one. In this case for determination of the image profile of 3D object's perfectly absorbing edge we first of all have calculated an impulse reaction of optical projecting diffraction limited system. 3D perfectly absorbing edge according to a model of equivalent diaphragms of our constructive theory [4] one can describe by two boundary step Heaviside functions: $f(x_1) = Y(x_1)$, $g(\tilde{x}_1) = Y(\tilde{x}_1)$ (Fig. 5). As a result 3D object under coherent illumination a spatial spectrum is described by the following equation:

$$F(\omega) = \int_{-\infty}^{+\infty} f(x_1)\tilde{g}(\tilde{x}_1 + \omega d/k)\, e^{-j\omega x_1} dx_1 = $$
$$= \int_{-\infty}^{+\infty} Y(x_1)\tilde{Y}_d(\tilde{x}_1 + \omega d/k)\, e^{-j\omega x_1} dx_1 \qquad , \quad (6)$$

where $\omega = k\,\theta$ ($k = 2\pi/\lambda$).

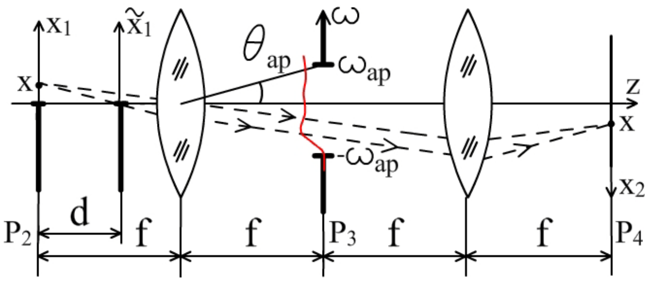

Fig. 5: The impulse reaction of the optical system (fragment of Fig. 1) for formation of 3D object's image in perfectly incoherent light

For calculation of the impulse reaction one place a point source in input plane P_2 with coordinate $x_2 = x$, i.e. $f(x_1) =$ $= \delta(x_1 - x)$, where δ(z) is the Dirak delta function.

As a result for $F(\omega, x)$ one can obtain:

$$F(\omega, x) = \tilde{Y}_d(x + \omega d/k)\, e^{-j\omega x} \qquad (7)$$

This field is filtered by the aperture diaphragm with angular size $2\omega_{ap} = 2k\,\theta_{ap}$. After this operation the field amplitude at $x_2 = 0$ in plane P_4 (geometrical position of 3D edge boundary) is equal to:

$$h_{coh}(x) = \oint_{-\omega_{ap}}^{\omega_{ap}} /\tilde{Y}_d(x + \ d \ k \ e^{-j\omega x} d\omega \quad (8)$$

Really $h_{coh}(x)$ is the impulse reaction for coherent system. To calculate the impulse reaction for perfect incoherent system one can fulfill module squaring: $h_{incoh}(x) = \left|h_{coh}(x)\right|^2$. Using $h_{incoh}(x)$ one can calculate the field intensity in $x_2 = 0$, when the input boundary function $f(x) = Y(x)$ is placed in plane P_2:

$$I(0) = \int_{-\infty}^{+\infty} Y(x)\, h_{incoh}(x) dx \quad (9)$$

Using Eq. (8) and Eq. (9) we have studied the 3D edge formation in incoherent light at weak volumetric effects ($\theta_{cr} >> \theta_{ap}$) and strong ones ($\theta_{cr} << \theta_{ap}$).

In first case the influence of the aperture on function $\tilde{Y}_d(x+\omega\ d\ k)$ is negligible amount due to small aperture size (this is similar to δ-function effect). Therefore, for intensity $I(0)$ one can obtain the following expression:

$$I(0) = \int_0^{+\infty} \left|\tilde{Y}_d(x)\right|^2 \frac{\sin^2(\omega_{ap} x)}{\pi \omega_{ap} x^2} dx \quad (10)$$

According to calculation the intensity $I(0)$ is determined by the following equation:

$$I(0) = 0.5 - 0.6 \frac{\theta_{ap}}{\theta_{cr}} \quad (11)$$

In case of the strong volumetric effects for the incoherent impulse reaction one can obtain from equation (8) after its integration by parts the following expression:

$$h_{incoh}(x) = \frac{\sin^2(\omega_{ap} x / 2 + kx^2 / 4d)}{\pi \omega_{ap} x^2} \quad (12)$$

It is important that at $d \to \infty$ ($\theta_{cr} \to 0$) a back face of 3D edge acts as a low-frequency rejecting filter which reduces the aperture in two times (active aperture size is ω_{ap}). The intensity $I(0)$ is calculated according to the following equations:

$$I(0) = \int_0^{\infty} \frac{\sin^2(\omega_{ap} x / 2 + kx^2 / 4d)}{\pi \omega_{ap} x^2} dx \quad (13)$$

It can be shown that $I(0)$ for this case is equal to

$$I(0) = 0.5 + 0.05 \frac{\theta_{cr}^2}{\theta_{ap}^2} \quad (14)$$

Using Eq. (11) and Eq. (14) one can correct the threshold levels at determination of boundary position of 3D edge under precision 3D measurements.

5. CONCLUSION

We have studied the peculiarities of image formation for 3D object as a thick edge with sharp shadow projection illuminated by the quasi-monochromatic partially space coherent illumination and perfectly incoherent illumination applied to dimensional inspection. The research results provide evidence that shift of 3D object shadow boundary depends on object's thickness, angular source size, and angular aperture size. It is shown that the choice of a source size can minimize the shift of real 3D edge position. We have developed the analytical equations for threshold algorithms for the cases of weak and strong object's volumetricity. The developed algorithms for processing the measurement information allow one by choosing the angular light source size to increase significantly the precision of 3D objects geometrical parameters measurement.

The obtained results can be used for development of precision measurement systems for 3D objects inspection with sharp shadow projection.

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