CALCULATION OF INDIRECTLY MEASURED RUNOUT OF BEVEL GEARS

Anke GUENTHER[1]

1 Hexagon Metrology GmbH, Wetzlar, Germany, anke.guenther@hexagonmetrology.com

Abstract:
The classic (direct) runout measurement is a function oriented all-over inspection of a bevel gear. Its result is one of the criteria to classify gears in standardised accuracy grades (former quality groups). For several reasons, this direct method is rarely applied for bevel gear inspections, carried out with coordinate measuring machines (CMM). Instead, special evaluations based on pitch measurements of both flanks (coast and drive) are commonly used. These calculations relate to much less flank information than the direct measurement and include, depending on the algorithm, several risks. This paper presents an improved approach for this evaluation and compares theoretical and experimental runout results carried out with several CMMs.

Keywords: bevel gear inspection, runout, CMM

1. INTRODUCTION

For the classification of gears regarding their quality, different measurement results are used. One classification criterion is the gear's runout deviation. The conventional and standardized runout measurement of a bevel gear is based on a simultaneous probing of both flanks of one gap by an appropriate probing tip at a well-defined pitch diameter near the reference cone. Spheres are the most common types of stylus tips, where the sphere diameter depends on the module and the number of teeth.

Since the introduction of coordinate measuring machines (CMM) this runout measuring principle lost its importance. Today, the runout evaluation of both cylindrical and bevel gears refers to the points obtained during pitch measurements [1]. In the case of cylindrical gears, this evaluation is based on mathematically well-defined flank surfaces and only occasionally gives reason for discussions.

In contrast to cylindrical gears, the flank surfaces of bevel gears are defined by a set of nominal points ([1][2][3]), adjusted in two grids covering both flanks of one tooth. Although all CMMs equipped with a bevel gear measurement software provide runout evaluations, only a few publications exist about appropriate algorithms. This paper presents a new approach for this evaluation problem and discusses further options [4]. Results of experimental investigations, carried out with different standard CMMs and discussed in section 2.7, verify this approach.

2. EVALUATION OF RUNOUT DEVIATION

2.1 Direct runout measurement

In [5] and [6] the runout measurement of bevel gears is defined as a radial position difference of a specified probe, which radially dips into each gap of the gear. The sampling direction of this probe is perpendicular to the reference cone and the probing sphere contacts both flanks simultaneously (Figure 1 a and b).

During of the sampling runout measurement the probe moves along the sampling cone. This sampling cone inspects the reference cone perpendicularly. The intersection element of these two cones is a circle with the so-called tolerance or pitch diameter. The sampling cone represents the totality of all possible positions and sampling movements of the probing sphere centre (Figure 1 b).

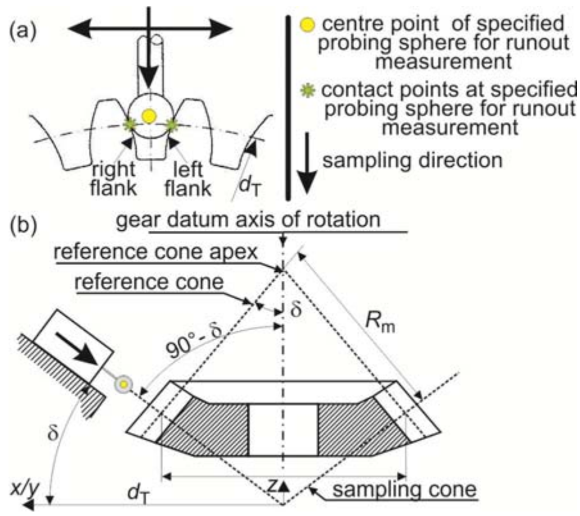

Figure 1: Definition of runout measurement conditions.
(a) Runout measurement via registration of one single point with a specified probing sphere, contacting both flanks simultaneously. Source: [7], edited.
(b) Movement direction of the probe, defining the sampling cone.
d_T: tolerance diameter (pitch diameter)
δ: cone angle (angle of reference cone)
R_m: mean cone distance (between apex of the reference cone and the intersection of the reference cone with the pitch diameter)

Figure 2 presents the common runout diagram with one bar per gap. The individual height of each bar reflects the varying dipping intrusion depths of the probing sphere relative to the depth in gap one. The diagram shows a typical runout deviation pattern of a gear with an eccentricity of 300 µm. The standardised runout parameter F_r ([5][6][7]) is defined as the span between the maximum and the minimum deviation (Figure 2).

As another convention for the evaluation of single runout deviations, Figure 2 b shows the same values, normalized to their mean value. The total runout deviation F_r in this diagram remains unchanged. The single values in Figure 2 b comply with distances to a concentric mean circle. If a sinus would be directly approximated to all deviations in Figure 2 b, the amplitude of the sinus would represent a first approach to the eccentricity of the gear.
Actually, a variation of this type of runout measurement, performed by manual probing with a simple dial indicator is applied for fast manufacturing inspections leads to a high measuring uncertainty.

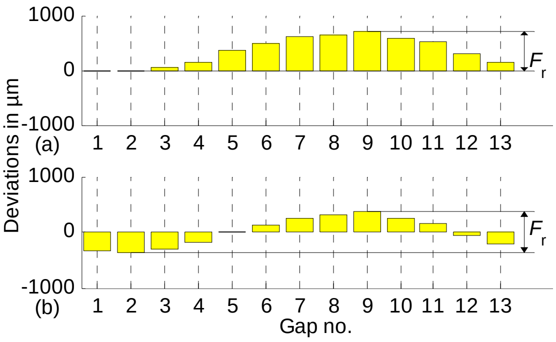

Figure 2: Diagrams presenting runout deviations.
(a) Runout single deviations in relation to a concentric circle intersecting the probing sphere centre point in gap 1.
(b) Runout single deviations from of Figure 2 a, normalized to their mean value.

2.2 Runout evaluation based on pitch measurement

The pitch of a bevel gear is defined in [5],[6] as the arc length between all consecutive left or right flanks of one gear, measured at the pitch diameter d_T in a distance R_m from the apex of the reference cone. This measurement position is equal to that of the runout. Therefore, CMM manufacturers tend to save measurement time by using the points obtained during the pitch measurement to evaluate the runout. Thus, the measuring strategy described in section 2.1 loses its importance. Figure 3 shows the principle of pitch measurements, where both probing movements sample pitch points at all left and right flanks, respectively.

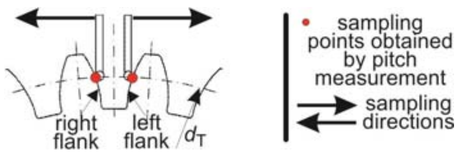

Figure 3: Runout measurement via the probing of two single points with a small probing sphere, corresponding to a pitch measurement. Source: [7], edited.

Figure 4 illustrates the difference between the sampling points obtained either by a pitch measurement or by a direct runout measurement of a spiral bevel gear. For the latter method, the contact points of the probing sphere with the left and the right flanks are on different *z*-positions, whereas the pitch points are on the same *z*-level. For helical cylindrical gears, this fact is less critical, because the involute flank form is mathematically well defined as a continuous surface. Thus, it is possible to determine exactly the positions of the probing sphere centres. For spiral bevel gears, the mathematical problem is not discussed in the literature and manufacturers of CMMs or software providers give no information about the implemented algorithms.

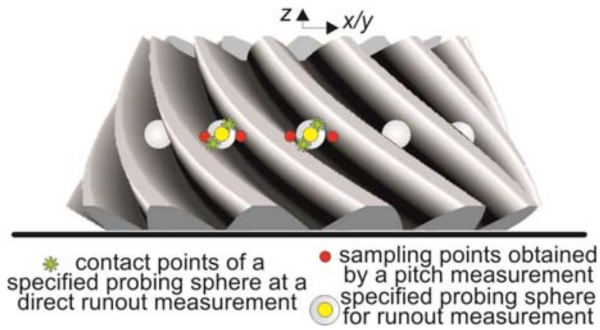

Figure 4: Positions of the probing sphere contact points during direct runout measurement, compared with the positions of the pitch sampling points

Moreover, since the *z*-positions of all drawn points are different (see Figure 4), a mathematical solution for the runout evaluation has to be able to determine the 3D positions of the (virtual) probing sphere centre based on the (measured) pitch points.

2.3 General explanations for runout evaluation

All software systems for the evaluation of measured bevel gear data, sold by CMM manufacturers, include an individual method for calculating the theoretical dipping depths of probing spheres, based on measured pitch points. Some of them imply the determination of eccentricity from runout results. Neither the implemented algorithms nor comparisons of measurement results carried out with different CMMs have been published, as yet. A reasonable accuracy assessment regarding the runout value F_r is problematical. Both, the ideal direct runout and the pitch-based runout should be measured at one gear in one clamping with one CMM. The subsequent evaluation based on pitch points should utilize the same probing sphere radius as used for the direct runout measurement. A comparison of these results would permit a fair decision. But, the pitch-based programs neither offer to choose the theoretical (virtual) probing sphere radius nor display the virtual sphere radius used during evaluation. Furthermore, the CMM control units often don't offer a convenient sampling strategy with a self-centring option, which is required for the direct runout measurement. Therefore, the desired comparisons between the two measurements are only possible on two different measuring devices with probably varying radii of the probing spheres.

Unlike cylindrical involute gears, a complete mathematical description of the bevel gear flanks is not available. Instead, the flanks of bevel gears are usually defined point wise. For the nominal left and right flank, a grid of 9x5 surface points exists together with a normal direction for each point. Consequently, it is required to estimate the runout deviations by iterative algorithms, based on diverse simplifications of the flank areas near the (measured) pitch and (virtual) runout measurement points. Thus, a precise analytical solution for a bevel gear runout evaluation is not possible.

2.4 Algorithm based on tangential planes

The pitch measurement points of both flanks reveal the distance of two opposite flanks of one gap. E.g. in a gap wider than the datum gap the probing sphere enters deeper. The only information content of a pitch measurement point is the position of the corresponding flank (more exactly: the flank area) in terms of its polar angle. Topographical deviations are still unknown.

As a first simplification, the angular position of a topographically ideal flank is given by the pitch deviation. In reality, the contact points of the probing sphere hardly ever coincide exactly with one of the given flank grid points. Therefore, the flank surfaces near the pitch points and the contact points have to be approximated, in order to estimate the position of a virtual probing sphere centre at double-sided contact.

The normal directions of the datum grid points contain additional information about the flank form. The second simplification is the assumption that the curvature of the flank close to a pitch point is relatively small. Thus, it is possible to approximate both flank regions by two tangential planes. These planes limit a theoretical corridor within the gap. The probing sphere contacts this corridor close to the pitch points.

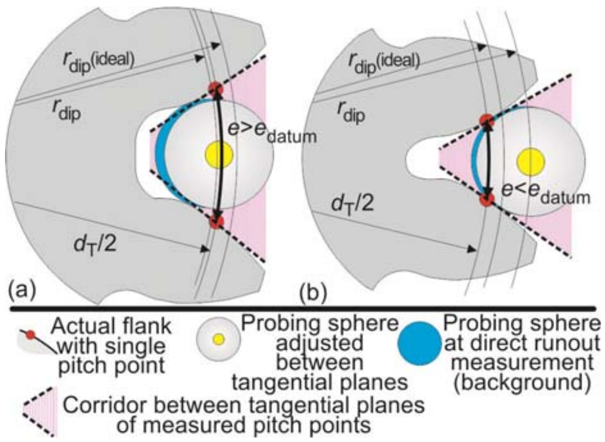

Figure 5: Corridors for diverse radii of dipped positions
(a) Situation in a wider gap, deeper dipped sphere.
(b) Situation in a narrower gap, sphere dipped-in less.
e_{datum}: Datum gap width
e: Actual gap width
d_T: Pitch diameter
r_{dip}: Radius of the probing sphere centre position, where a simultaneous contact at both flanks occurs

The effect of this corridor in cases of wider and narrower gaps are shown in Figure 5 a-b. They illustrate the effect of the second simplification. The background (blue 'crescent of the moon') images the probing sphere during direct runout measurement. In both cases, the estimated position of the probing sphere (in the foreground) is not deep enough inside the gap. But, this systematic error influences the estimated value of r_{dip} in each gap in a similar way, i. e. the runout parameter F_r, defined as the maximum relative deviation (span) of r_{dip} between any two gaps, in less affected than the individual r_{dip} values.

Finally, the described corridor has a three-dimensional geometry. For bevel gears, this gap typically gets wider towards the tooth tip and the back of the reference cone and narrower towards the tooth root and the apex of the reference cone. Thus, one could imagine the corridor as a conical slit, where the probing sphere is rolling through. Consequently, the corridor centre line indicates all theoretically possible positions of the probing sphere centre. This model reflects the reality only in a limited region around the pitch points. Within this region, the intersection point between the sampling cone and the corridor centre line gives the virtual probing sphere centre during direct runout measurement (see also Figure 1 b).

2.5 Estimations based on curved elements

For a better estimation of the probing situation inside a gap, standard geometric elements with one, two or more curvatures offer better geometry models. Investigations with several standard geometries show that e.g. a torus fits better than a cylinder [4]. [8] exclusively used several tori for his investigations. On the other hand, the more complex the description of the geometric element is, the higher is the mathematical effort regarding the evaluation of the virtual probing sphere centre points. Compared to the tangential plane model, an approximation with cylinder segments showed significant improvements, as results obtained by a direct runout measurement showed [4].

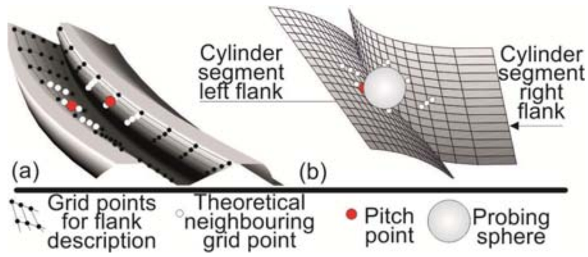

Figure 6: Approximation of flank regions near to the pitch points by cylinder segments.
(a) Bevel gear gap, defined by two sets of grid points, with indicated pitch points and their neighbouring points.
(b) Mathematical gap, formed by two cylinder segments approximating the two pitch points and their neighbouring points, together with a probing sphere for the runout evaluation.

2.6 Algorithm based on cylindrical segments

The objective of the following estimation is to create an improved mathematical gear model, compared to the tangential plane corridor. To approximate two cylinders describing the two investigated flank areas it is recommended to include all direct and indirect neighbouring grid points around the pitch points, as shown in Figure 6 a. Thus, the approximation of a suitable cylinder is based on nine grid points. The pitch point is commonly given by the point in the middle of the whole grid set of one flank. Close to the pitch points both cylindrical segments in Figure 6 b represent the gap with a sufficient accuracy by a curved slit. In order to determine the virtual positions of probing spheres for double-sided contacts, one can substitute the inspected surfaces by equidistant surface models, shifted by one sphere radius in normal direction. By this, it is possible to determine deviations directly as the distance between the substitute surface model and the probing sphere centre point. The same idea applied to cylindrical segments results in an intersection line between two cylindrical segments, which represents all possible positions of the probing sphere centre in this gap. Again, intersection with the sampling cone leads to the virtual probing sphere centre position, used for runout evaluation.

2.7 Experimental results

To complete the investigations on runout evaluations, a test was carried out with a pinion gear described by the following parameters: number of teeth z=17, module m_t=4.3 mm, spiral angle β=45° and angle of the reference cone δ=23.6°. This gear was measured on 5 different CMMs, manufactured by 4 different CMM producers. The test measurements comprised the geometry features topography, pitch and tooth thickness. The runout evaluation was based on the pitch measurement. Additionally, the measured points were exported for further investigations.

The results in topography deviations varied over a range of 10µm. The reasons are different evaluation methods and diverse references [2][3]. The results concerning the tooth thickness and the total (cumulative) pitch deviation F_p varied over a range of 1.5 µm, belonging to the accuracy grades 2 and 3 [6]. These results show that all CMMs sampled very similar measurement points. Therefore, it was expected that the runout results would be very similar too.

But, the values of the runout deviations F_r varied between 6.2 and 13.4 µm. This corresponds to accuraccy grades from 2 to 4, which is obviously not satisfying. For a comparison measurement, a special CNC-program was implemented to measure the runout directly. 10 measurements at reproducibility conditions delivered runout deviations between 5.2 and 7.2 µm. This complies with a classification in accuracy grade 1 and 2. As a result one has to note that just one CMM evaluated a reasonable runout, compared to the direct measurement.

In order to investigate this result in detail, the exported measured points were evaluated with the runout algorithm according to the tangential plane model (Section 2.4). The results for F_r varied between 9.1 and 10.3 µm in a very small range. This complies with an accuracy grade 3. The same points evaluated according to cylindrical segment model (Section 2.6) result in runout deviations F_r varying between 6.0 and 7.3 µm, again in a very small range. This meets the accuracy grade 2. This result can be interpreted as a confirmation that an algorithm based on gap models with curved elements leads to better estimations than the simplified algorithm referring to tangential planes.

3. SUMMARY

Generally, the algorithms for runout evaluations based on pitch measurements have a significant influence on the characteristic value F_r. Geometry models with curved approximated flanks like cylinders or tori estimate F_r better than simpler models. The investigation of several runout evaluations based on pitch measurements gave some insights into commercially available CMM software systems:

a) Obviously, some CMM runout routines calculate higher deviations compared to the direct runout measurements. This results in an underrated quality classification, which possibly causes higher costs for the production of bevel gears.

b) The inspection strategy named 'back to direct measurement!', would enable a more function-oriented evaluation with the advantage that topographic deviations are considered. But this contradicts to the intended reduction of measurement times. Furthermore, modern gear processes with specialized bevel gear manufacturing machine tools work very uniformly such that significant variations of the flank topographies at one gear are decreasingly probable. These tests discussed in this paper were carried out with only one pinion gear. Since this experimental basis is much too small, more research work and further investigations are necessary. Generally, a certification of bevel gear evaluation software, comparable to the certification of software for cylindrical gears [9], appears reasonable.

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