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TOLERANCE DESIGN AND ITS APPLICATIONS – CASE STUDIES

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Abstract: *In today's competitive marketplace, companies are under increased pressure to produce products that have low cost and high quality. Product cost and quality are influenced by many factors. One factor that strongly influences both is manufacturing variation. No manufacturing process can produce a part with perfect geometry. In modern manufacturing, tolerance design principles are adopted to manage the manufacturing variation and its effect on product performance and manufacturing cost. This paper presents an overview of tolerance design principles adopted in the design of mechanical assemblies. Some typical case studies on tolerance design are presented to illustrate the tolerance design concepts applicable to practical problems.*
Keywords: *Tolerance design, tolerance analysis, tolerance allocation, cost-tolerance cost, mean shift, truncated tolerance.*

1. INTRODUCTION

During the manufacturing process, component's dimensions are affected by a number of variables including the structure of the process machines, wear of the machining tool, skill levels of operators, precision of the inspection instruments, measurement environment and variations in materials. Thus, it is impossible to produce components with exactly the same dimensions. The variation exists because no production process is perfect. Often times, controlling this variation is attempted during production when substantial effort and resources, e.g., time, money, and manpower, are required. Manufacturing variation is the range of values that a product's dimensions assume during manufacturing. The range of variation specified (permitted) in design is called tolerance. According to the American Society of Mechanical Engineers (ASME 1994) tolerance is the total amount by which a specific dimension is permitted to vary from a nominal value. The tolerances are assigned to components to account for the dimensional and form variations.

Design engineers would like to assign narrowed tolerances, so that functional satisfaction of products can be ensured. However, manufacturing engineers usually prefer to use wider tolerances. Because achieving tight tolerances during production needs substantial effort and resources, e.g., time, money, and manpower, are required. Wider tolerances imply that the difficulty of producing parts is minimized. Thus the manufacturing cost of products will be reduced. The proper amount of tolerance assignment is trade-off between these factors. Tolerances also greatly influence the selection of production processes by process planners and determine the assemblability of the final product. Tolerance specification, then, is an important link between engineering and manufacturing.

1. 1. Tolerance design

Assemblies consist of many components that interact with each other to perform a task. The function of a product is generally controlled by a critical (assembly) dimension in the assembly. The critical dimension depends of individual dimensions involved in the dimensional chain. These dimensions are called functional dimensions. Any variation in the functional dimensions will directly affect a critical dimension. The functional dimensions' tolerances generally accumulate (stack up) and affect the performance of the assembly; poor performance, inability of assembling components, rejection/rework of the assembly, etc. depends on the variations in each of the component parts in the assembly. Engineers know that tolerance stacking or accumulation in assemblies controls the critical clearances and interferences in a design, such as lubrication paths or bearing mounts, and thus affects their performance. The assigning tolerance to the components and controlling the accumulation of tolerance in an assembly for the product quality and cost is called tolerance design (Ramani, et al. 1998). Tolerance design is divided into two areas: tolerance analysis and tolerance synthesis or tolerance allocation.

2. TOLERANCE ANALYSIS

It is the process of estimating the accumulation of the design tolerances on component dimensions and features to ensure that parts will assemble during production.

Tolerance analysis attempts to identify the sources of variation, how the tolerances assigned to components propagates through the system, and suggest adjustments to minimize the final product variation. Modeling the tolerance analysis is a good step to understand the tolerance stack up and designing products that are unaffected by tolerance stack up. Modest efforts in this area can yield significant cost savings with little capital investment (Nigam and Turner, 1995). Many researchers have investigated the tolerance stack up to minimize final cost and quality loss of a product while it is being designed. The following tolerance analysis models were found in literature (Chase et al, 1991; Hong and Chang, 2002).

1. Worst case (WC) model
2. Root sum square (RSS) model
3. Modified RSS model
4. Estimated mean shift model
5. Monte Carlo simulation model

The first two tolerance accumulation models were first used by Fortini (1967) who written a book that summarizes this analysis model. They are the most commonly used tolerance stack up models for tolerance analysis/allocation. The salient points of these models are discussed in the following sub sections.

2. 1. Worst case model

Traditionally, tolerance analysis has been studied in the worst case (WC) context (Wade, 1967). It assumes all errors are at their extreme values at the same time. As per worst case method, the errors accumulate linearly with the number of parts. This method is simple, but it may cause unnecessarily tight tolerance specifications, which leads to an increase in the cost of manufacturing. The WC model is given in equation (1).

$$dU = \sum \left[\left| \frac{\partial Y}{\partial X_i} \right| t_i \right] \leq t_a \quad (1)$$

Where, X_i are the nominal component dimensions, t_i are the component tolerances, dU is the predicted assembly variation, t_a is the specified limit for dU. The partial derivatives ($\partial Y/\partial X_i$) represent the sensitivity of the assembly tolerance to variations in individual component dimensions, X_i. For one-dimensional tolerance stack up the sensitivity is ±1.0.

2. 1. 1. Assumptions and risks of using WC model

In the worst-case approach, the designer's only assumption is that all components are within the tolerance limits. While this may not always be true, the method is so conservative that parts will probably still fit. The major disadvantage of the worst case model is, when there are large numbers of components or a small assembly tolerance, the worst case model yields small tolerances for components, which will be costly.

2. 2. RSS model

Statistical tolerance calculations based on normal distribution were proposed by Mansoor (1963). Experience in manufacturing indicates that small errors are usually more numerous than large errors. The RSS model assumes that the manufactured dimensions fit a statistical distribution called a normal curve. The errors accumulate with square root of number of parts. This model also assumes that it is unlikely that parts in an assembly will be randomly chosen in such a way that the worst case conditions will occur. Statistical tolerancing is cheaper because looser tolerances are allowed at each part or feature (cost rises as tolerances are tightened). Statistical process control and statistical tolerancing permit a bet on interchangeability. The RSS model is given in equation (2).

$$dU = \sum \left[\left| \frac{\partial Y}{\partial X_i} \right| t_i \right]^2 \leq (t_a)^2 \quad (2)$$

2. 2. 1. Assumptions and risks of using RSS model

The RSS model yields larger component tolerances for a given assembly gap, but the risk of defects at assembly is higher. The RSS model assumes that component tolerances are tied to process capabilities. All process distributions are centered on the midpoint of the dimension. The RSS model is better than the WC model as it accounts for the tendency of components to be centered on a mean dimension. In general, this model is not used if there are less than four dimensions in the tolerance stack up (Creveling, 1997).

2. 3. Modified root sum of the squares model

The empirical studies have shown that the RSS model does not accurately predict what is manufactured because some (or all) of the RSS assumptions are not valid. The designers can use the RSS model with a "correction" factor and the new model is called the modified root sum of the squares method (MRSS). Spotts (1983) report the modified RSS model as,

$$[t_a]^2 \leq C_f \sum \left[\left| \frac{\partial f}{\partial x_i} \right| \cdot t_i \right]^2 \quad (3)$$

Where, C_f = Correction factor used in the MRSS equation.

Several experts have suggested correction factors (C_f) in the range of 1.4 to 1.8 (Mansoor, 1963, Chase and Greenwood, 1988). Historically, the most common factor is 1.5.

2. 3. 1. Assumptions and risks of using the MRSS model

The uncertainty associated with the MRSS model is that there is no mathematical reason for the factor C_f. The correction factor can be thought of as a safety factor. The more the RSS assumptions depart from reality, the higher the safety factor should be. It applies the same safety factor to all the tolerances, even though they don't deviate from the RSS assumptions equally.

2. 4. Estimated mean shift model

Greenwood and Chase (1987) introduced the mean shift model to account for the shift of mean due to setup error or drifts due to such time varying factors such as tool wear. They blend the worst case and RSS models to account the less knowledge about the processes for manufacturing or vendor supplied part. The estimated mean shift model is given by:

$$t_a \le \sum |m_i t_i| + \sqrt{\sum \left((1-m_i)(t_i)^2\right)} \qquad (4)$$

Where, m_i = the mean shift factor for the i^{th} component.

In this model, the mean shift factor is a number between 0.0 and 1.0 and represents the amount that the midpoint is estimated to shift as a fraction of the tolerance range. If a process were closely controlled, we would use a small mean shift, such as 0.2. If we know less about the process, we would use higher mean shift factors. The first part of the Estimated Mean Shift model is the sum of the mean shifts and is similar to the worst case model. Notice if we set the mean shift factor to 1.0 for all the components then assembly tolerance t_a is equal to assembly tolerance as predicted by the WC model. The second part of the model is the sum of the statistical components. Notice if we used a mean shift factor of zero for all of the components then t_a is equal to assembly tolerance as predicted by the RSS model. The two major advantages of the estimated mean shift model are:

- It allows flexibility in the design. Some components may be modeled like worst case, and some may be modeled statistically.
- The model can be used to estimate designs (using conservative shift factors), or it can accept manufacturing data (if it is available).

2. 5. Monte Carlo simulation model

The Monte Carlo simulation remains the most widely used method for the statistical analysis of mechanical assemblies, due to the versatility and accuracy. The Monte Carlo method performs assembly simulations using a random number generator, which selects values for each manufactured variable, based on the type of statistical distribution assigned by the designer. These values are combined through the assembly function to determine a series of values of the assembly variable. This series is then used to find the first four moments of the assembly variable. Finally, a distribution matching the four moments can be used to determine the mean, standard deviation, and percentage of assemblies which fall outside the design specifications or assembly rejections. The Monte Carlo method procedure is shown in Fig. 1 (Chase and Parkinson, 1991).

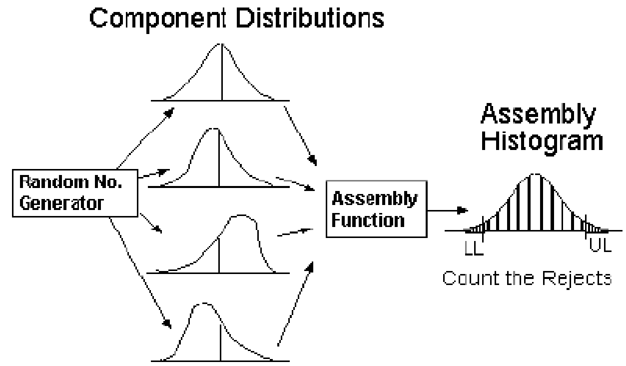

Fig. 1 Assembly tolerance analysis by Monte Carlo simulation

Monte Carlo Simulation can be used for both nonlinear assembly functions and non-normal distributions. The biggest disadvantage of the Monte Carlo method is that in order to get accurate estimates it is necessary to generate very large number of samples, and it is computationally intensive. The number of simulated assemblies must be of the order of 100,000 to 400,000 to predict the small percentage rejects of modern manufacturing processes (Chase and Parkinson, 1991). Also, if the distributions of the independent variables change, the whole analysis must be redone. The Monte Carlo Simulation can be performed using software like Crystal Ball, based on the assumed probability distribution for the individual parts.

3. TOLERANCE ALLOCATION / TOLERANCE SYNTHESIS

Tolerance allocation is a design tool for reducing over-all cost of production, while meeting the target levels for quality [Wade, 1967]. Using tolerance allocation tools, a designer may distribute the assembly tolerance among the components of the dimensional chain, systematically by tightening tolerances on less expensive processes and loosening tolerances on costly processes, for a net reduction in cost. The tolerance allocation tries to complete or augment the tolerance specification, originally made from experience or empirical knowledge, by incorporating some heuristic, optimization, and/or other methods. Most of the approaches that have been published are based on the optimization of the cost-tolerance function. They usually try to get optimal tolerance `values' while the tolerance `types' are assumed as fixed. The following methods are available in literatures to perform the tolerance allocation by Chase and Parkinson (1991):

1. Proportional scaling
2. Cube root of the nominal
3. Difficulty factors
4. Minimum cost criterion
5. Minimum cost with process selection

The salient aspects of minimum cost criterion and minimum cost with process selection are briefly indicated below.

Minimum cost criterion: If an empirical equation of cost vs. tolerance (or process capability) can be obtained for each dimension in the assembly sum, then an optimization algorithm may be used to systematically search for the combination of component tolerances which results in the least overall production cost.

Minimum cost with process selection: Optimization procedures have been extended to not only find the least cost set of tolerances, but to also select the least cost process from a set of alternative processes for each dimension for the assembly (Chase et al, 1989).

3. 1. Minimum cost tolerance allocation

As per the design requirement, the tolerance should be as near zero as possible. If the tolerances are too tight the manufacturing cost becomes high, while too loose tolerances give less manufacturing cost, but they are associated with poor performance. Thus the tolerance allotment becomes an optimization problem having constraints as either specified yield or manufacturing cost. The specified yield denotes the acceptance probability of a component or an assembly after tolerance allotment. The tolerance design literature emphasizes the use of optimization methods to minimize the manufacturing cost and sensitivity. The sensitivity of a particular dimension is the change in the critical dimension for the unit change in the dimension. In most of tolerance optimisation problems, the assembly manufacturing cost is reduced by adjusting the tolerance on individual components such that the costly tolerances are reduced while the less expensive tolerances are tightened as shown in Fig. 2.

4. TOLERANCE ANALYSIS VS. TOLERANCE ALLOCATION

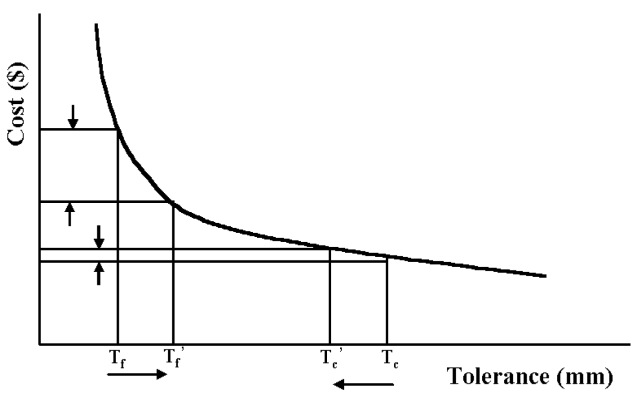

Fig. 2 Tolerance-cost relationship and suggested tolerance change strategies

Tolerance analysis and tolerance synthesis can be seen as complementary constraint satisfaction methods after the tolerances have been specified (refer Fig 3). A central issue in tolerance specification is that engineers are more commonly faced with the problem of tolerance allocation rather than tolerance analysis. In tolerance analysis, the component tolerances are all known or specified and the resulting assembly tolerance is calculated. In tolerance allocation, on the other hand, the assembly tolerance is known from design requirements, while the component

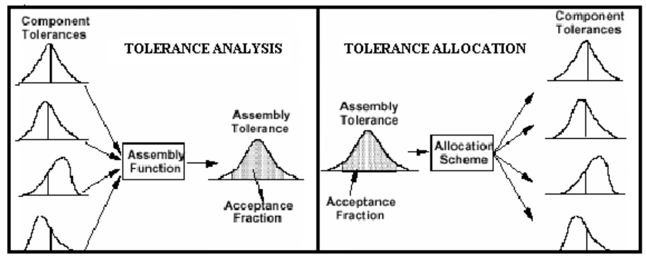

Fig. 3 Tolerance analysis vs. Tolerance allocation (Chase, 1999)

tolerances are unknown.

Tolerance allocation is a design function. It is performed early in the product development cycle, before any parts have been produced or tooling ordered. It involves first, deciding what tolerance limits to place on the critical clearances and fits for an assembly, based on performance requirements; second, creating an assembly model to identify which dimensions contribute to the final assembly dimensions; third, deciding how much of the assembly tolerance to assign to each of the contributing components in the assembly. Tolerance analysis, on the other hand, is a production function. It is performed after the parts are in production. It involves first, gathering data on the individual component variations; second, creating an assembly model to identify which dimensions contribute to the final assembly dimensions; third, applying the measured component variations to the model to predict the assembly dimension variations.

5. COST TOLERANCE MODELS

Allocating the component's tolerances effectively to satisfy simultaneously the functional requirement of products and the least manufacturing cost possible has been always the object of research. A necessary factor in optimum tolerance allocation is the specification of cost-vs.-tolerance functions. The tolerance costs are calculated on a per part basis. When tighter tolerances are called for, speeds and feeds may be reduced and the number of passes increased, requiring more machining time leads to higher manufacturing costs. In practice, the cost tolerance data are normally obtained from the machine shop through experiments or observations and recorded in the form of discrete points. He (1991) proposed a single process of cost estimation model to evaluate the overall production cost. He divided the production cost into material cost, manufacturing cost and scrap cost. The manufacturing cost was further divided into machining cost, rework cost and inspection cost (refer Fig 4). Rework cost, inspection cost and scrap cost are considered as variable costs. When the designated tolerance is smaller than the process capacity, the scrap cost and rework cost increase as well the number of inspection items required. However, during multiple processes operation, it is necessary to select a suitable

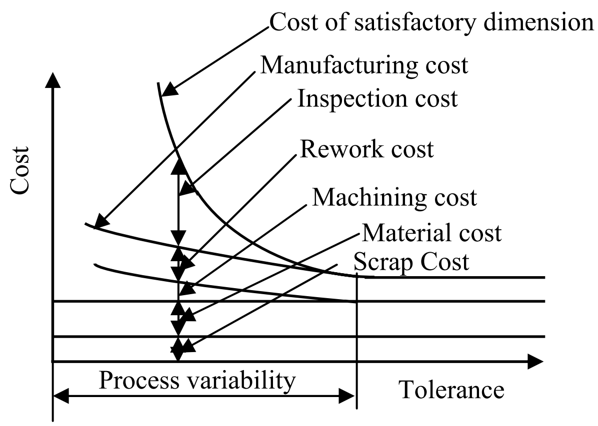

Fig. 4 Cost components of a machining process

process based on the designated tolerance of component's dimensions.

The best combination of the interrelated design tolerances, which satisfies the constraint of the given assembly tolerance and leads to the least total production cost are considered. Many researchers have focused on the mathematical modeling of cost-tolerance relations and the optimization of related tolerances for minimum production costs. Speckhart (1972) and Spotts (1973) used the Lagrange multiplier method to minimize the manufacturing cost. Wilde and Prentice (1977) used geometric programming to minimize the exponential manufacturing cost. Ostwald and Huang (1977) applied the 0-1 integer programming approach to solve the deterministic tolerance problem and its variations. Lee and Woo (1989) proposed a branch and bound algorithm that uses linear programming to solve a large-scale deterministic tolerance problem. Dong and Soom (1992) presented an empirical cost-tolerance data for the typical production process including turning on lathe, face milling, drilling and hole machining, rotational surface and internal grinding as well as die sand and investment casting. The authors also presented new tolerance-cost models and a hybrid model tolerance optimization.

The manufacturing cost–tolerance characteristics are machine specific and therefore selection among the alternatives whether the manufacturing processes or the machines is similar, and thus leads to the same strategy of problem formulation and solution. Michael and Siddall (1981) introduced the powers and exponential hybrid models. Chase and Greenwood (1988) introduced the reciprocal model with better empirical data fitting capability. A discrete cost tolerance model and the tolerance optimisation method using reliability index were introduced by Lee and Woo (1989). This method directly utilizes the empirical data, thus having no curve fitting errors. Wu et al. (1998) studied various existing continuous cost tolerance models and compared their modeling errors. Dong et al. (1994) compared these tolerance-cost models based on model fitting errors and proposed three Hybrid models and three polynomial models. The various cost tolerance models are presented in the literatures are listed in Chase and Parkinson (1991) and Dong et al. (1994).

A study of cost vs. tolerance for the various metal removal processes over the full range of nominal dimensions has been done by Gunasekaran et al (2004). The data collected has been curve fit to obtain reciprocal cost-tolerance function. They have also reported that the cost of manufacturing the required tolerances in Indian manufacturing industries are comparatively less when compared with the tolerance cost data reported by Chase et al. (1999). The tolerance cost data are collected from Indian industries involved in job-shop production. It is assumed that the US dollar $, is equal to 45 Indian Rupees. The comparison of tolerance cost data clearly indicates that tolerance cost is nearly eight to ten times lesser in Indian industries (see Table 1). The lesser manufacturing cost in India are due to lower machine hour rate, overhead, labour charges, low machine tool and other related production and inspection equipments cost, etc.

Table 1 Increased cost of manufacturing in US industries compared to Indian industries (In terms of number of times)

Grinding	Milling	Turning	Reaming	Lapping
9	8.5	10	8.5	9.5

6. TOLERANCE CHAIN

Tolerance analysis and synthesis requires a tolerance chain or a dimensional chain (assembly function). Bjorke (1989) mentioned in his book that the tolerance chain is the link formed among the interrelated tolerances of components. The tolerance chain describes the relationship between a user specified sum dimension and individual dimensions called functional dimensions of a given assembly. The tolerance chain is formed by identifying the sequence of dimensions such that the adjacent component surfaces are in contact with one in previous and one with successor component in the sequence. The mathematical representation of the tolerance chain provides a fundamental tolerance stack equation, which can be used to determine the critical dimension involved in the tolerance stack up.

In complex assemblies consisting of many components, the identification of functional dimensions and driving the relationship between critical and functional dimensions is a challenging task. Consideration of any additional dimensions in the tolerance chain for tolerance analysis will increase the functional dimensions to have smaller tolerances lead to higher manufacturing cost and reduction of assembly yield. Ramani, et al. (1998) proposed a dimensional block diagram (DBD) algorithm for automatic generation of tolerance chain. Wang, et al. (1993) suggested an algorithm for the automatic tolerance chain formation using mating relations between the components in contact. The mating information stored with the components is used to form a mating graph. Then the algorithm searches from one end of critical dimensions to other side and links the components to form the chain. Zhang and Porchet (1993) suggest an object functional relationship graph (OFRG) to represent the functional relationship between assembly components including the functional features like cylinders and faces.

6. 1. Dimensional Block Diagram (DBD) approach

The DBD algorithm was developed for the purpose of dimension normalization. It quickly identifies the functional dimensions and relationship between them and critical dimensions. The DBD algorithm has failed to generate tolerance chain in the following two cases: (i) Multiple dimensions with same starting point and (ii) Requirement of reversal in direction of tolerance loop.

6. 2. Modified DBD algorithm approach

The DBD algorithm was modified to overcome the above limitations and to perform tolerance analysis incorporating true position tolerances (Vishnukumar, et al. 2002). The MDBD is also able to draw the tolerance loop below the 2D AutoCAD drawing. The major steps in the MDBD algorithm are:

- Open the 2D assembly drawing and pick the dimension lines
- Extract the staring and end coordinates of dimesion lines and arrange them is ascending order of starting co-ordinates
- Form a list containing coordinates of dimension lines in accending order
- Form sublists by connceting the dimension’s end point with stating point of next dimension line
- Compare the footers and and equate the same footers to form tolerance chain
- If tolerance chain is not formed, then indentify the sublist’s footers and form footers sublists and identify the footers sublist footer. Eqaute the footers sublists footers to generate the tolerance equation
- Generate tolerance function equation and draw the tolerance loop below the 2D drawing.

Multiple dimensions with same starting point: The sub-assembly of butterfly valve shown in Fig. 5 is used for the illustration. The critical dimension exists only in one path out of all possible paths. The modified algorithm moves along all the paths available from a particular point in an assembly, and forms additional sublists, which are helpful to identify the tolerance chain. The tolerance chain formed by MDBD is given below the butterfly valve diagram and is shown in Fig. 5.

Reversal in direction of tolerance loop: The MDBD algorithm takes into consideration the reversal of the loop. The MDBD algorithm forms sub-lists and footers are identified. Then footers’ sub-lists are formed by comparing

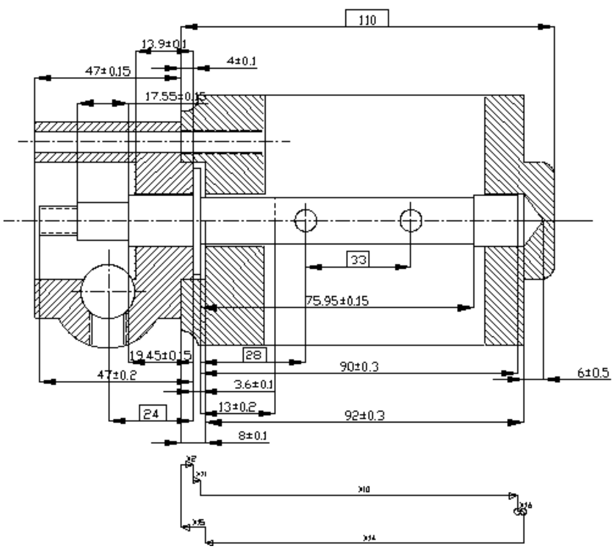

Fig. 5 Butterfly valve sub-assembly with tolerance loop

both the starting and ending values of each footer. If a dimensional chain is not formed from this sub-lists, then sub-lists of footers’ is formed and they are used to identify the tolerance chain like in regular DBD algorithm. The tolerance chain formed by MDBD is drawn below the

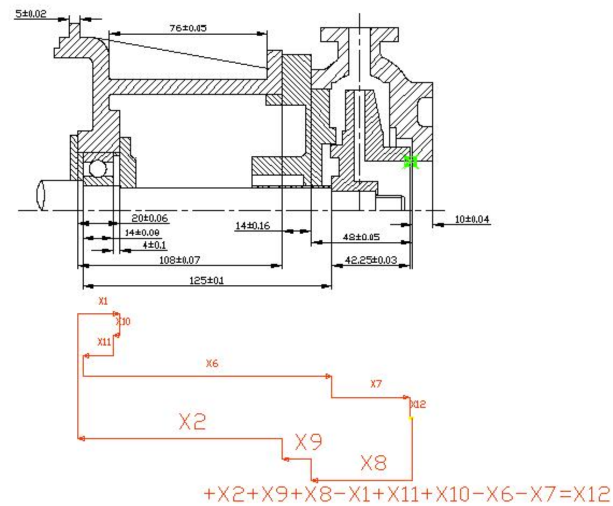

Fig. 6 Monoblock pump assembly with tolerance loop

monoblock pump assembly diagram and is as shown in Fig. 6.

7. CASE STUDIES IN TOLERANCE DESIGN

A monoblock pump assembly is used here to illustrate the tolerance chain formation using MDBD algorithm, Tolerance analysis and tolerance optimization methods. The monoblock pump consists of two group components; namely stationary housing components and rotating rotor assembly. The main criterion of the assembly is that the impeller marginal (radial) centerline and volute radial centerline should lie in the same plane or in different planes within the specified tolerance limit. The gap between the casing and the impeller is also critical with respect to the performance (hydraulic slip) of the pump. It decides the performance of the pump. The critical dimension is controlled by individual dimensions of the assembly as shown in the Fig 7. The critical dimension is specified as 0.75±0.3 mm.

7. 1. Tolerance chain formation

When the program is initiated, the MDBD algorithm extracts the starting and end coordinates of dimension lines when the dimension lines are picked manually one by one.

Table 2 List of dimensional coordinates in ascending order

Dimension	Dimension Value	Tolerance Value	Starting Point	Ending Point
X1	20	0.06	48.34	68.34
X2	108	0.07	48.34	146.54
X3	5	0.02	49.34	43.84
X11	14	0.08	50.54	65.04
X6	125	0.10	50.54	170.54
X10	4	0.10	65.04	68.34
X4	76	0.05	139.54	63.34
X9	14	0.16	146.54	160.54
X8	48	0.05	160.54	209.04
X7	42.25	0.03	170.54	208.04
X12	-	-	208.04	209.04
X5	10	0.04	209.04	219.30

Once all dimension lines are picked the program prompt for horizontal or vertical dimension tolerance analysis. After this the corresponding dimensions list are arranged in the ascending order of the starting coordinates (Table 2). Then sub-lists are formed (Table 3), analyzed and finally the tolerance loop and tolerance equation are displayed under the 2D pump assembly drawing as given in Fig 6. The footer C1 and C2 are equal, the dimensions sum of both can be equated to get the tolerance equation, *i. e.*

$$X1-X10-X11+X6+X7+X12+X5 = X2+X9+X8+X5$$
$$X1-X10-X11+X6+X7+X12 = X2+X9+X8$$
$$X12 = X2+X9+X8-X1+X10+X11-X6-X7 \quad (5)$$

Where, X12 is the critical dimension. The tolerance loop is drawn blow the diagram and tolerance equation is displayed in the drawing.

7. 2. Tolerance analysis

7.2.1 Worst case analysis

The critical dimension in the monoblock pump assembly is given in equation (5). The individual dimensions which are affecting the critical dimension are:

X2 = 108±0.07 mm X9 = 14±0.16 mm
X8 = 48±0.05 mm X1 = 20±0.06 mm
X10 = 4±0.10 mm X11 = 14±0.08 mm
X6 = 125±0.10 mm X7 = 42.25±0.03 mm

Tolerance of the critical dimension $t_a = \Sigma t_i$, Where t_i = Tolerance of the i^{th} dimension. The tolerance on critical dimension,

$t_a = \pm$ (0.07+0.05+0.10+0.10+0.16+0.06+0.08+0.03)

$t_a = \pm$ 0.65 mm

Table 3 Sub-lists formed for Monoblock pump assembly by MDBD

Sub List C1			Sub List C2			Sub List C3			Sub List C4		
X1	48.34	68.34	X2	48.34	146.54	X3	49.34	43.84	X11	50.54	65.04
-X10	68.34	65.04	X9	146.54	160.54				X10	65.04	68.34
-X11	65.04	50.54	X8	160.54	209.04				-X1	68.34	48.34
X6	50.54	170.54	X5	209.04	219.30				X2	48.34	146.54
X7	170.54	208.04							X9	146.54	160.54
X12	208.04	209.04							X8	160.54	209.04
X5	209.04	219.30							X5	209.04	219.30
Footer 48.34>219.30			**Footer 48.34>219.30**			**Footer 49.34>43.84**			**Footer 50.54>219.30**		
Sub List C5			Sub List C6			Sub List C7			Sub List C8		
X6	50.54	170.54	X10	65.04	68.34	X4	139.54	63.34	X9	146.54	160.54
X7	170.54	208.04	-X1	68.34	48.34				X8	160.54	209.04
X12	208.04	209.04	X2	48.34	146.54				X5	209.04	219.30
X5	209.04	219.30	X9	146.54	160.54						
			X8	160.54	209.04						
			X5	209.04	219.30						
Footer 50.54>219.30			**Footer 65.04>219.30**			**Footer 139.54>63.34**			**Footer 146.54>219.30**		
Sub List C9			Sub List C10			Sub List C11			Sub List C12		
X8	160.54	209.04	X7	170.54	208.04	X12	208.04	209.04	X5	209.04	219.30
X5	209.04	219.30	X12	208.04	209.04	X5	209.04	219.30			
			X5	209.04	219.30						
Footer 160.54>219.30			**Footer 170.54>219.30**			**Footer 208.04>219.30**			**Footer 209.04>219.30**		

7. 2. 2. Root sum square method

Tolerance of the critical dimension t_a = SQRT $(\Sigma t_i)^2$, where t_i = Tolerance of the i^{th} dimension. The tolerance on critical dimension is

$t_a = \pm\sqrt{(0.07^2+0.05^2+0.10^2+0.10^2+0.16^2+0.06^2+0.08^2+0.03^2)}$
$t_a = \pm\ 0.252$ mm

7. 2. 3 Monte Carlo simulation

The Monte Carlo simulation is performed using Crystal Ball Software. The nominal dimensions and their standard deviations (t_i/3) of the parts involved in the tolerance loop are given as input. The distributions of the individual components dimensional errors are also give as input. Figure 7 shows the input parameters. The software settings were done based on the input parameters and the type of result output required like parameter values, simulation updates during simulation runs, sensitivity, percentage contribution, etc. The number of simulation runs also selected. Figure 8 shows the distribution of critical dimension and distribution parameters. The critical dimension is obtained after 1,00,000 simulation runs as 0.75 ± 0.252 mm. The results obtained all three tolerance analysis methods are compared and given in Table 4.

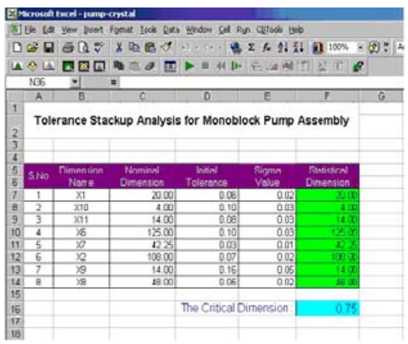

Fig. 7 Crystal Ball software inputs

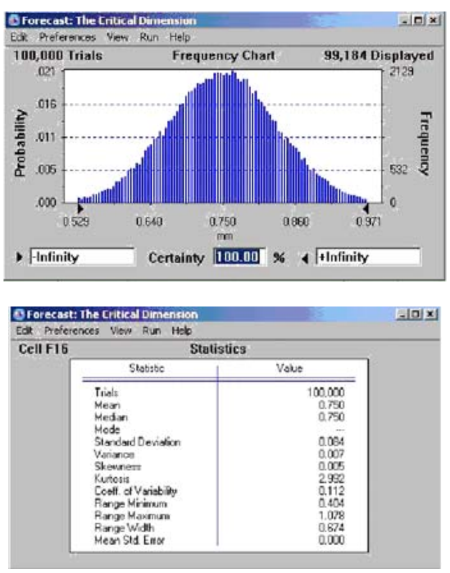

Fig. 8 Monte Carlo simulation results using Crystal Ball software

Table 4 Results Comparison

Tolerance summing method	Worst case analysis	Root sum square method	Monte Carlo simulation (1,00,000 trials)
Results in mm	0.75±0.650	0.75±0.252	0.75±0.252

7. 3. Tolerance optimization

The tolerance optimization is done using the Crystal Ball® 2000 – OptQuest® software which is simple to use. The nominal dimensions and their tolerances, and cost tolerance relationship of all the individual dimensions were given as input. The software calculates the initial assembly tolerance as 0.252 mm. The corresponding manufacturing cost was calculated to be $ 12.85 using the reciprocal tolerance cost function and the relevant cost data given by Chase (1999). Then the permissible variation in the critical dimension is also given as input. The software randomly varies the tolerance of individual component dimensions for each simulation run under the constraint that the resultant critical dimension should not exceed the permissible variation. For each simulation run the software calculates the critical tolerance and the corresponding manufacturing cost and finally arrives at the tolerances for individual components. The software relaxes tolerances where there is substantial reduction in cost is possible and tightens tolerances where the increase in cost due to tightening is not substantial. After the simulation the tolerance on critical dimension is widened from 0.252 mm to 0.300 mm, which reduces the manufacturing cost from $12.85 to $11.70. This shows the cost saving is obtained if the tolerance assembly dimension is widened. The simulation result was shown in Fig. 9 and 10 and is also given in Table 5.

Table 5 Optimized tolerance values

Dim. Name	Nominal value (mm)	Initial		Final	
		Tol. (mm)	IT Grade	Tol. (mm)	IT Grade
X1	20	0.06	IT9	0.03	IT8
X10	4	0.10	IT11	0.03	IT9
X11	14	0.08	IT10	008	IT10
X6	125	0.10	IT9	0.20	IT10
X7	42.25	0.03	IT7	0.05	IT8
X2	108	0.07	IT8	0.14	IT10
X9	14	0.16	IT12	0.05	IT9
X8	48	0.05	IT8	0.10	IT10
Total Assy. Cost ($)		12.85		11.70	

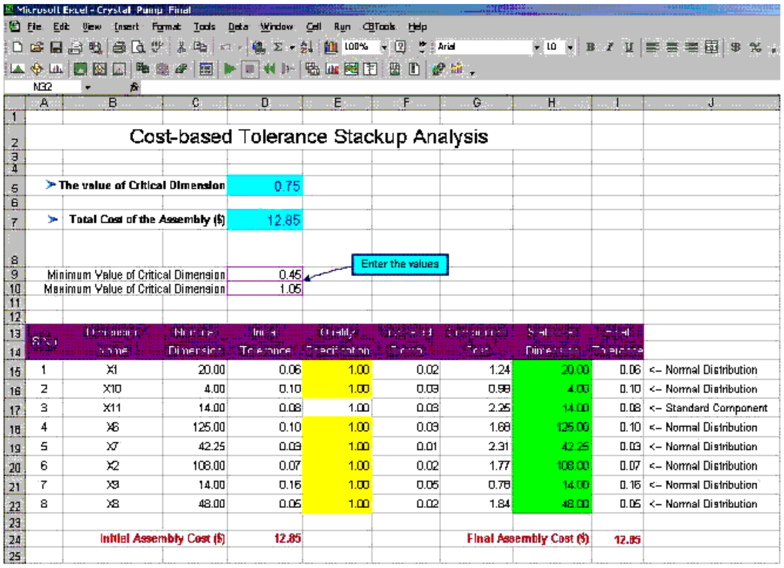

Fig. 9 Initial simulation with assembly manufacturing cost as $12.85

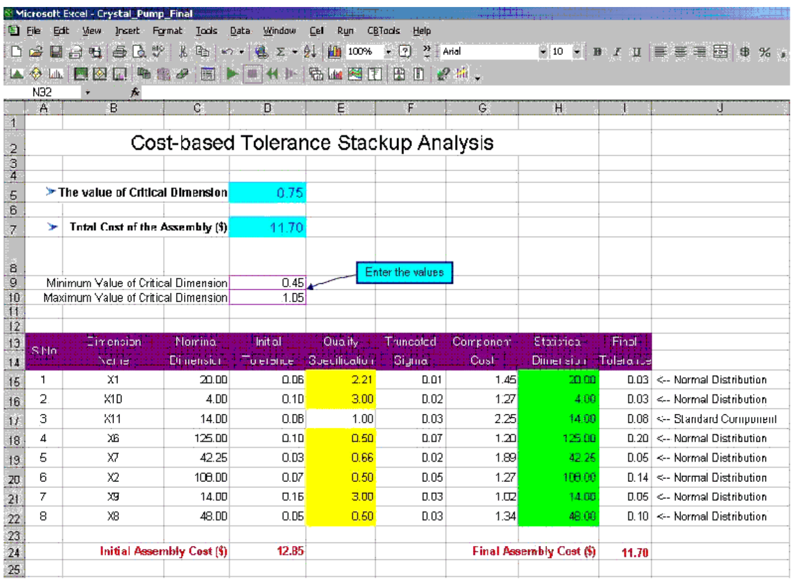

Fig. 10 Reduction of assembly manufacturing cost as $11.70

8. CONCLUSION

The tolerance design principles like tolerance analysis, tolerance allocation, cost - tolerance relationship, minimum cost tolerance allocation, controlling process variation parameters (mean shift and process variation) were discussed in this paper. The contribution of authors in this area also discussed. Some typical case studies are presented to illustrate the tolerance design methods discussed.

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