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An Analysis of Block Sizes for Compressive Sensing Reconstruction Applied in Image Processing Optimisation

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Abstract **– Signal sampling is a fundamental process of data acquisition systems and several studies have emerged regarding sampling methods following on Nyquist Theorem. Compressive Sensing (CS) proposes sampling of sparse signals with sub-Nyquist sampling rates. In short, CS is composed of a sampling/compress stage and a reconstruction stage. Some algorithms are used where this last. One of these is the Orthogonal Matching Pursuit (OMP) where 2D Discrete Cosine Transform (2D-DCT) can be used. This work compared the application of 2D-DCT transform and CS theory on images as either a whole or split in blocks. As a result, the influence of block size is revealed using the Mean Square Error (MSE) metric for different block sizes.**

Keywords – Compressive Sampling, Image Compression, DCT, Block Size Influence, Innovation.

I. Introduction

Most natural phenomena can be considered analog, *i.e.*, they can show any real value at any point in time. The human voice is a common example of an analog signal [1]. Digital data acquisition, which is a fundamental part of Instrumentation, and topics related to electronics, microcontrollers, communications, automation, etc., consists of generating digital signals that can be processed and comprehended from a physical quantity, as shown in Fig 1. The Analog-to-Digital Converter (ADC) is the device used for converting an analog signal into digital [2].

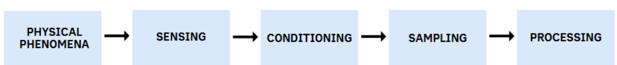

Fig. 1. Data Acquisition Flow.

Analog-to-Digital data conversion must follow the Nyquist Theorem, which states that the minimum sampling frequency, called the Nyquist rate, must be at least twice the maximum signal frequency in order to avoid aliasing [3].

A novel theory called Compressive Sensing, or Compressive Sampling (CS), works below the Nyquist rate. CS theory asserts that an analog signal can be converted into digital with fewer samples than what the Nyquist-based sampling determines. CS takes advantage of the redundancy that certain signal classes present in nature, in particular, CS works well in "sparse" signals, where most of its samples are close to zero and only a few samples concentrate the signal information [4]. A K-sparse signal indicates a signal with sparsity K, i.e. with K non-zero coefficients.

In another context, a digital image is a matrix (a two-dimensional (2D) array) of pixels represented as a $N \times N$ vector or in a NN sequential concatenated array, as shown in Fig. 2.

Fig. 2. Transmission of an image as a vector

In the medical field, notable recent CS works include magnetic resonance imaging (MRI) images compression [6], electrocardiogram (ECG) monitoring [7], and ultrasound images compression [8]. In the image scope, some notable CS works involve microwave imaging application [9], 3D thermoacoustic imaging [10], infrared and visible image fusion [11], radar target recognition [12], and hyperspectral image fusion [13]. For image communications, more specifically with block divisions and DCT compression, notable recent works can be seen in [14-16].

In this work, we have been observing the influence of applying the 2D Discrete Cosine Transform (2D-DCT) on image blocks and in the reconstruction times for different block sizes. For other types of images, such as thermal, microwave imaging, etc., for example, other transform basis may be more adequate instead of the 2D-DCT.

The remainder of this paper is organized as follows. Section II presents the background on compressive sensing. An analysis of block sizes, as well as the reconstruction algorithm used in this work, are presented in Section III. Results and discussions are shown in section IV and conclusions are made in section V.

II. Compressive Sampling

Consider a discrete-time signal $x \in R^N$, as shown in (1), where $\Psi \in R^{N \times N}$ represents a transform basis and can be called "sparsity basis", and $s \in R^N$, is the sparse representation of x, *i.e.* with most of its coefficients close to zero.

$$x = \Psi s \quad (1)$$

The basic system of CS consists of taking $y \in R^M$ measurements of x, with K < $M << N$, as in (2), where $\Phi \in R^{M \times N}$ is called the "measurement matrix" and it is associated with a set of linear measurements of x.

$$y = \Phi x = \Phi \Psi s \quad (2)$$

The measurement matrix must allow the reconstruction of x from y, and it must also be incoherent with the sparsity basis Ψ. Coherence, or low incoherence, can be set down as the maximum correlation between any two elements of Ψ and Φ, i.e. the higher inner product value of any two elements of Ψ and Φ, as in (3). If Ψ and Φ are incoherent, then the information contained in x will be uniformly spread through all coefficients of y. Thus, every coefficient will have part of the global information of x, and only $M < N$ samples are required to recover x [5].

$$\mu(\Phi, \Psi) = \sqrt{N}\, max\{\Phi^T \Psi\} \quad (3)$$

The reconstruction is done by solving the optimization problem in (4), where $||\cdot||_1$ denotes the l_1-norm given by the sum of the magnitudes of s, and $\Theta = \Phi\Psi$.

$$min_{s \in R^N} ||\hat{s}'||_1 \ subject\ to\ \Theta s = y \quad (4)$$

One reconstruction algorithm is the Orthogonal Matching Pursuit (OMP) where 2D Discrete Cosine Transform (2D-DCT) can be used to transform a signal from a base to a sparse-base.

III. Analysis of Block Sizes

The reconstruction step is responsible for recovering the original image representation from generated measures. Considering M, the measurements vector size, and N, the original vector size, it results in a set of equations with M equations and N variables. As the CS proposal is to generate a system with $M << N$, the set of equations has fewer equations than variables. There are some ways to solve this problem, which include the Orthogonal Matching Pursuit (OMP) algorithm.

OMP is a type of greedy algorithm, which is a step-by-step iterative method. In this approach, the estimation $\hat{s}$ is updated by selecting only the columns of Θ that are highly correlated with the M measurements y [17]. OMP executes the following steps:

1. Define the initial residual as y
2. Find the "atom" (the column that maximally improves $\hat{s}$), with the highest correlation with the residual
3. Update the estimation $\hat{s}$
4. Update the residual by an orthogonal method
5. If it reaches a termination criterion, it stops. Otherwise, it searches for the next best atom and repeats steps 2-5.

For non-sparse images, a transform basis can be applied to it and take it to a sparse domain. A popular transform basis used is the DCT, which groups together the data contained in some vectors to the first elements of the set. In the case of images, the 2D-DCT is used, given that pixels have a correlation with neighboring pixels and not only linearly as a vector. The inverse 2D-DCT can also be used in the reconstruction.

Fig. 3 shows the image Lena and its sparse representation on the right, with all of its information concentrated in the upper left corner.

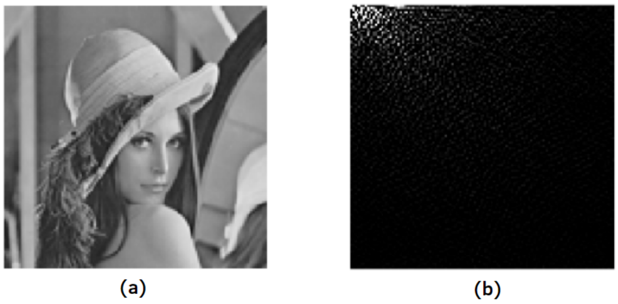

Fig. 3. DCT result (b) applied to the image Lena (a).

After the DCT is applied, the image is ready to be sampled. A test was performed for a 128 × 128 image, resulting in a vector with size $N = 16384$. The objective is to sample a vector of size $N = 8192$, resulting in a 50% compression of the original image and it's shown in Fig. 4.

It can be seen that the reconstruction was completed, but the resulting image presents some errors. An improvement to this problem is applying the DCT in blocks of the image, as shown in Fig. 5. Thus, the application of DCT, de CS method, and the reconstruction are executed block by block.

The reconstruction time using blocks is much lower than the regular method. Fig. 6 shows the image reconstruction with 50% compression.

Knowing that applying CS block by block results in more efficient reconstruction, the next step is to find the influence of the block size in reconstruction.

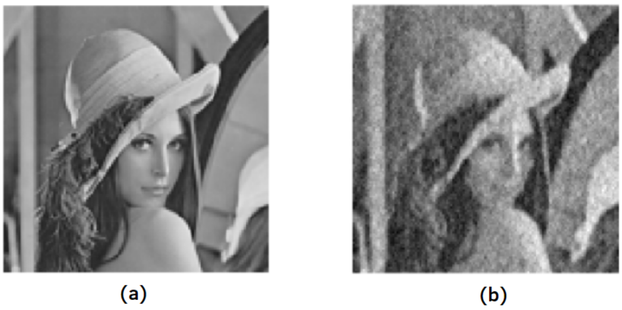

Fig. 4. CS result (b) applied to the image Lena (a).

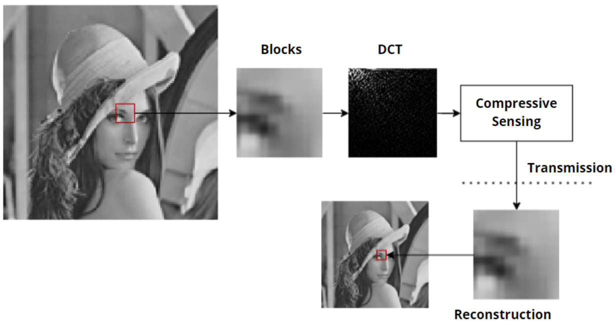

Fig. 5. CS method flow applied block by block

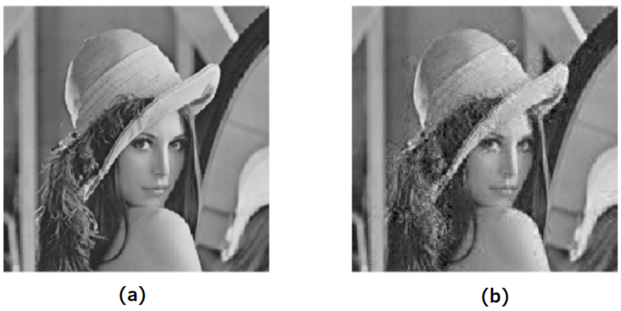

Fig. 6. Original image in (a) and image reconstructed block by block with CS shown in (b).

IV. RESULTS AND DISCUSSIONS

Considering the proposed improvement of block-by-block application for 2D-DCT, the influence of block sizes in the reconstruction process was evaluated using the Orthogonal Matching Pursuit method.

The evaluation is measured from two main parameters: reconstruction time and mean square error between the original and reconstructed image.

Thus, the sweeping of block size was executed for the Lena image with size 512 × 512, varying block sizes for 4 × 4, 8 × 8, 16 × 16, 32 × 32 and 64 × 64 pixels. The results of reconstruction time can be seen in Fig. 7.

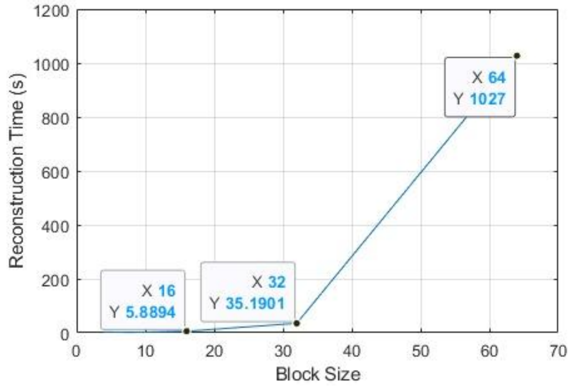

Fig. 7. Reconstruction times for different block sizes

The increase in block size requires more time to find a matching solution. In this implementation, the OMP termination condition is the sparsity, which was defined as one-fourth of the measurements vector y size. Hence, increasing the block size results in more needed measurements and higher time at each iteration, although fewer iterations are needed.

The other evaluated parameter was the reconstruction time, as shown in Fig. 8. Using smaller blocks generates high mean square errors. As mentioned in Section III, the DCT and sampling are executed with blocks of an image taken from the original image. If smaller-sized blocks are used, the DCT results contain less information about said blocks, so it generates a high error value. On the other hand, increasing block size shows an error value stabilization.

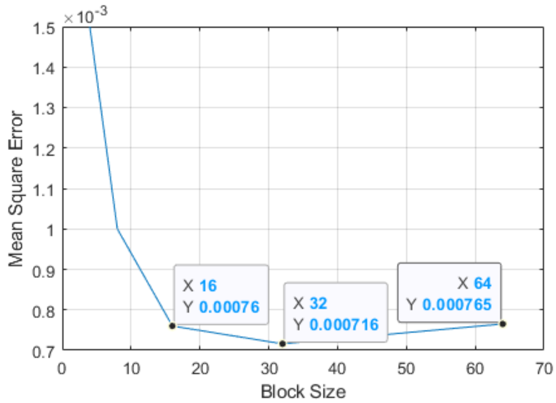

Fig. 8. MSE for different block sizes

Considering the results in Fig. 7 and Fig. 8, it is necessary to take into account the trade-off between error and processing time. Block sizes of 16 and 32 were chosen for comparison. The first has a slightly higher error, but a reconstruction time of about one-seventh of the time when compared to the latter. The result of compressive sensing with blocks 16 × 16 is presented in Fig. 9. The results in Fig. 8 must be looked at only as a comparison between methods, given that the tests were not executed in a time-optimized environment.

Fig. 9. Original image in (a) and image reconstructed with 16's block shown in (b).

V. Conclusions

In this paper, the main concepts of compressive sensing were presented, the reconstruction of an image was executed in blocks and the reconstruction time and MSE were analyzed. Optimal block size can be defined when considering the used parameters and it generates a significant improvement in reconstruction results. This improvement is shown in achieved results in generated images and graphs where, as a result, it is necessary to have a trade-off between error and processing time, that is, the larger the block size, the less the MSE, but the higher the processing time.

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