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A B-spline and OS-ELM Fusion Approach for Prognostics with Singularity Problems

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***Abstract* – In practice, the degradation process of electronic products is usually accompanied with singularities caused by intermittent and transient interference, improper conduct on singularity processing would inevitably and seriously affect the accuracy of products' life-time prediction. Taking advantage of rapid development of AI technology recent year, a new surrogate approach based on spline function and online sequential extreme learning machine (OS-ELM) is developed in this paper, to address these issue. This fusion approach takes the cubic non - polynomial spline function as the prediction cell of the output from OS-ELM, the second derivative of the spline model can be adopted and calculated to form a series of observation frames, meanwhile, an improved particle swarm optimization (PSO) is used to optimize the parameters of ELM hidden network to help with forecasting the observation sequence and rebuild the spline function. In the verification stage, two numerical simulation examples and a practical application involving typical time series data with singularities demonstrate the effectiveness of this proposed fusion method, respectively.**

Keywords – Prognostic; Fusion; Spline; OS-ELM; Singularity; PSO.

I. INTRODUCTION

In modern industry, electronic equipments are more and more widely used. Unlike the slowly aging phenomenon of traditional mechanical systems, the failure of electronic systems usually includes intermittent and transient interference. These singularities of intermittent and transient interference phenomenons are likewise usually regarded as noise in traditional solutions. However, they contain many early signs of failure and other hidden infoemation, sometimes, they will even be transmitted to every node of electronic systems, causing uncertainty propagation. Thus, the improvement of singularity signal processing is actually valuable and non-negligible for enhancing prediction accuracy.

Due to development of powerful AI technology these years, AI based predictive maintenance has become more efficient. Therefore, aiming at the features of electronic systems mentioned-above, AI based real-time prediction schemes should be introduced to develop online reliability prognostic scheme that can update more comprehensive monitoring data on time, instead of long-term forecasting.

This paper is organized as following: Section III introduces relevant algorithm architecture, then, continues to exhibit the detail of cubic non-parameter spline model and PSO improved OS-ELM algorithms. In Section IV, two simulation studies and a practical application case are respectively conducted to demonstrate the good quality of this proposed time series forecasting method. Section V concludes this whole paper.

II. RELATED RESULTS IN THE LITERATURE

A B-spline function is a piecewise defined polynomial function with several beneficial properties such as numerical stability of computations, local effects of coefficient changes and built-in smoothness between neighbouring polynomial pieces [1], thus, it can be seen as an approximation surrogate model. A common application of B-spline functions, curves and surfaces is fitting of data points. Fitting can either be interpolation or approximation. An interpolating B-spline function passes through the data points. In this paper, an interpolating B-spline function model and an online single-hidden layer feedforward neural networks (SLFNs) are developed for assessing products' reliability performance in relation to the prediction of their unique and key data.

Online Sequential Extreme Learning Machine (OS-ELM) [2-3] is a sequential variant of extreme learning machine (ELM). To efficiently and effectively deal with problems with sequential data, it is able to learn data one-by-one with fixed or varying chunk size. It leads to a great deal of popularity recently because of its extremely fast learning speed and has been implemented in numerous methods and applications.

However, OS-ELM also has some shortcomings :1) Due to new sequential data adding, the network structure given in advance might no longer be able to achieve the expected error;2) some distribution of parameter in the hidden layer might be randomly selected, which would cause unstable network output [4-7].

Due to the inherent flaw of OS-ELM, and considering the actual situation factors, training time and prediction accuracy, this paper adopts a Quantum Behaved Particle Swarm Optimization (QPSO) algorithm [8] to optimize the parameters of OS-ELM in the initial stage to ensure the prediction accuracy. QPSO is easy to realize, has few parameters and good global convergence ability, which all could help it optimize the initial parameters and get into the next stage of online adjustment.

Then a fusion method combined with particle swarm optimization (PSO) is going to estimate the relevant parameter, in order to update the spline model. The objective is to improve the forecasting accuracy when the degradation data contains some singular points. In contrast to previous studies, this novel method does not need to identify the singular data in the whole forecasting process, but still shows a perfect performance even if there exist some singular points in signal.

III. DESCRIPTION OF THE METHOD

1. Spline-OS-ELM Model Description

In this section, we propose a spline cell-based OS-ELM model and combined with QPSO algorithm, in this ensemble mathematical model, the prediction cells are based on the cubic non-polynomial spline with a 2-order smoothness.

The framework of this fusion prediction model and the prediction process are given, in Figure 1, where the effect of singularity in the degradation data determined the choice of spline approximation model. The Particle swarm optimization method is used to improve and optimize the output of hidden layer from OS-ELM. In the end, a combined model is obtained, to update the spline cell predictor, help forming the most likely spline model.

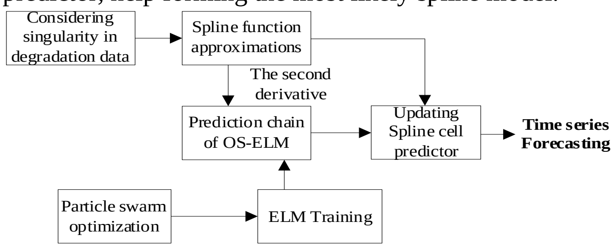

Fig. 1. The framework of Spline-OS-ELM Model

2. Spline function approximations

The non-polynomial spline used in this method is a trigonometric-polynomial spline function, and its infinitely differentiable triangular part can make up for the deficiencies of finite smoothness of a polynomial. Hence, for a spline approximation, this method can give a better fitting result as compared with other methods.

For $i = 0,1,\cdots,n,$ we discretize the interval (a,b) by equal distance such that $x_i = a + ih$, where $x_0 = a, x_n = b,$ h=(b-a)/n, and n is a positive integer. Then, for each sub-interval $[x_{r-1}, x_r]$, $r = 1,\cdots,n,$ the cubic non-polynomial spline $s_r(x)$ is defined as the following form[9]:

$$s_r(x) = a_r \cos(x - x_{r-1}) + b_r \sin(x - x_{r-1}) + c_r(x - x_{r-1}) + d_r \tag{1}$$

where a_r, b_r, c_r and d_r are spline coefficients. Let $w = \theta \cdot h$, we can obtain the coefficients of equations (1) as follows from a series of algebraic operations:

$$\begin{cases} a_r = -h^2 \dfrac{M_r - 1}{w^2} \\ b_r = -h^2 \dfrac{M_r - M_{r-1}\cos w}{w^2 \sin w} \\ c_r = -h^2 \dfrac{M_r - M_{r-1}}{w^2} - \dfrac{y_{r-1} - y_r}{h} \\ d_r = h^2 \dfrac{M_r - 1}{w^2} + y_{r-1} \end{cases} \tag{2}$$

where $y_r = s_r(x_r)$, $M_r = s_r^{(2)}(x_r)$, and here x_r is a positive integer which represents the data. In addition, we can calculate the parameter θ by the following equation:

$$\begin{cases} \alpha = -(\dfrac{1}{w^2} - \dfrac{1}{w \sin w}) \\ \beta = \dfrac{1}{w^2} - \dfrac{\cot w}{w} \\ \alpha + \beta = 0.5 \end{cases} \tag{3}$$

Using the continuity of first and the second derivative at (x_r, y_r) , we can further obtain the following relation[10-11]:

$$h^2(\alpha M_{r-1} + 2\beta M_r + \alpha M_{r+1}) = y_{r-1} - 2y_r + y_{r+1}, \quad r = 1,2,\cdots,\mathrm{n}-1 \tag{4}$$

So it follows that

$$\alpha M_{r-1} + 2\beta M_r + \alpha M_{r+1} = \frac{y_{r-1} - 2y_r + y_{r+1}}{h^2} = \frac{1}{h}\left(\frac{y_{r+1} - y_r}{h} - \frac{y_r - y_{r-1}}{h}\right) \tag{5}$$

Let $d_r = \frac{1}{h}(\frac{y_{r+1} - y_r}{h} - \frac{y_r - y_{r-1}}{h})$ then

$$\alpha M_{r-1} + 2\beta M_r + \alpha M_{r+1} = d_r, \quad r = 1,2,\cdots,\mathrm{n}-1 \tag{6}$$

Further, the boundary condition is determined by the following formulas:

$$2\beta M_0 + \alpha M_1 = \frac{6}{h}\left(\frac{y_1 - y_0}{h} - y_0\right) = d_0 \tag{7}$$

where

$$y_0 = \frac{-y_2 + 4y_1 - 3y_0}{2h}$$

Combining equations (6) with equations (7), it can be written in the form of a matrix as follows:

$$\begin{bmatrix} 2\beta & \alpha & & & \\ \alpha & 2\beta & \alpha & & \\ & & \cdots & & \\ & & \alpha & 2\beta & \alpha \\ & & & \alpha & 2\beta \end{bmatrix} \begin{bmatrix} M_0 \\ M_1 \\ \vdots \\ M_{n-1} \\ M_n \end{bmatrix} = \begin{bmatrix} d_0 \\ d_1 \\ \vdots \\ d_{n-1} \\ d_n \end{bmatrix} \tag{8}$$

The second derivative vector $M = (M_0, M_1, M_2, \ldots, M_n)$ which can be solved by equation (2) will be taken as a vector frame of time series and be inputted into this Spline-OS-ELM model, in order to generate a new vector M and $s_r(x)$ by forecasting model.

These are the outcome of a data analysis based on the second order derivative which corresponds to the singularity, which is represented as the second order derivative of the predicted deviations. It follows, that the bank of data encapsulated in the collection of windows are utilized in window-based manner for predicting the future behavior.

3. Online Sequential Extreme Learning Machine

The learning process of OS-ELM algorithm for output weight could be divided into two parts: Part 1 is the initial stage, where the initial output weight is obtained through a small number of samples; Part 2 is the online learning phase, in which data blocks composed of samples were put into the network and output weights are updated.

The OS-ELM Network structure [4-7] is shown in Fig. 2:

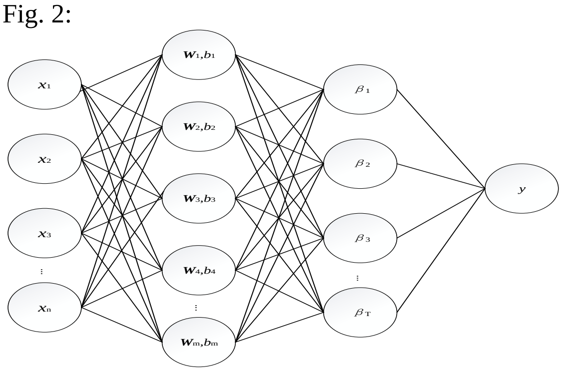

Fig.2. The structure of online sequential extreme learning machine

The ELM structure is specifically composed of the following three parts:

Input layer: Containing real-time data information with predictive objects $x=\{x_1, x_2,\dots x_i, \dots,x_n\}$, where x_i is the data input from the ith dimension.

Hidden layer: Consisting of hidden layer nodes in the neural network $(W, b)=\{(W_j, b_j)\}$, where W_j is the weight of the jth node, b_j is the bias coefficient of the jth node. Applying Sigmoid as the active function hereto obtain $h_j=G(W_j, b_j, x)$.

Output layer: the output layer weight can be expressed as $\boldsymbol{\beta}=\{\beta_1,\beta_2,\dots, \beta_k,\dots, \beta_L\}$, where β_k is the kth weight of the output layer corresponding to the hidden layer output.

Let t_i is the true value of the ith sample, $\boldsymbol{T}=[t_1,t_2,\cdots,t_n]$. The calculation method of hidden layer weight is obtained:

$$\boldsymbol{\beta}=(\boldsymbol{H}^{\mathrm{T}}\boldsymbol{H}+\lambda \boldsymbol{I})^{-1}\boldsymbol{H}^{\mathrm{T}}\boldsymbol{T} \quad (9)$$

$\boldsymbol{H}$ is the matrix composed of outputs for each sample hiding layer:

$$\boldsymbol{H}=\begin{bmatrix} G(W_1, b_1, x_1) & \dots & G(W_L, b_L, x_n) \\ \dots & \dots & \dots \\ G(W_1, b_1, x_n) & \dots & G(W_L, b_L, x_n) \end{bmatrix} \quad (10)$$

Let $\boldsymbol{K}_0$ is intermediate matrix, $\boldsymbol{K}_0=\boldsymbol{H}_0^T\boldsymbol{H}_0$; $\boldsymbol{K}_0^{-1}$ is the inverse matrix of $\boldsymbol{K}_0$, $\boldsymbol{F}_0=\boldsymbol{K}_0^{-1}$.

At online sequential learning phase,the output weights are updated whenever a new chunk of input data with N_{k+1} training sample arrives, the output sample of the hidden layer $\boldsymbol{H}_{k+1}$, the output weight $\boldsymbol{\beta}_{k+1}$ and $\boldsymbol{F}_{k+1}$ could be obtained by the following recursive formula:

$$\boldsymbol{\beta}_{k+1}=\boldsymbol{\beta}_k+\boldsymbol{F}_{k+1}\boldsymbol{H}_{k+1}^T(\boldsymbol{T}_{k+1}-\boldsymbol{H}_{k+1}\boldsymbol{\beta}_k) \quad (11)$$

$$\boldsymbol{F}_{k+1}=\boldsymbol{F}_k-\boldsymbol{F}_k\boldsymbol{H}_{k+1}^T(\boldsymbol{I}+\boldsymbol{H}_{k+1}\boldsymbol{F}_k\boldsymbol{H}_{k+1}^T)^{-1}\boldsymbol{H}_{k+1}\boldsymbol{F}_k \quad (12)$$

Through these equations, it can be deduced that, in OS-ELM algorithm, the pre-given network structure may no longer be appropriate, the disadvantage of random selection of hidden layer parameters would lead to unstable network output.

4. Quantum behaved Particle Swarm Optimization

Aiming at the above problems, this paper adopts QPSO algorithm [8] to optimize the threshold and input weight in the initial stage of OS-ELM, so that a more reasonable structure and a smaller error can be guaranteed in the initial stage of the network.

QPSO algorithm utilizes quantum behaved characteristics of particles to search the solution space, each individual would be described by a particle in quantum space to reflect the uncertainty of particles. The unique memory function of particle swarm makes them could dynamically track the current situation and adjust the search strategy, which have strong global search ability and robustness.

Defining particle as the possible solution (C,σ) in the optimal solution space, adopting (root mean square error, RMSE) as the fitness function of PSO algorithm:

$$Fitness(i)=RMSE=\sqrt{\frac{1}{n_{test}}\sum(y_{test}-y_{pred})^2} \quad (13)$$

Where n_{test} is the number of testing samples, y_{test} is the true value of testing sample and y_{pred} is the predicted output value of this model.

The training steps for QPSO to optimize ELM parameters are as follows:

1) Calculating the fitness value *Fitness (1)* of the first particle x_1, making the optimal fitness value *Fitness (best) = Fitness (1)*, and take particle x_1 as P_{best},
2) Input weights w_i and hidden layer bias are b_i randomly initialized to construct the ELM neural network structure, and a set of data (C,σ) is randomly generated as the initial solution space,
3) Adopting optimized ELM model to train corresponding samples, calculating and evaluating the fitness value. Each particle (C,σ) could adjust its own position and velocity by comparing the fitness value of the individual extremum and global extremum,
4) Determining whether the iteration times are reached. If so, the algorithm is terminated, and the

search results are given. Otherwise, keep iterating to get a new particle swarm,

5) Finally, Establishing the optimized ELM model based on the optimal (C,σ).

The detailed algorithm description is shown in figure 3.

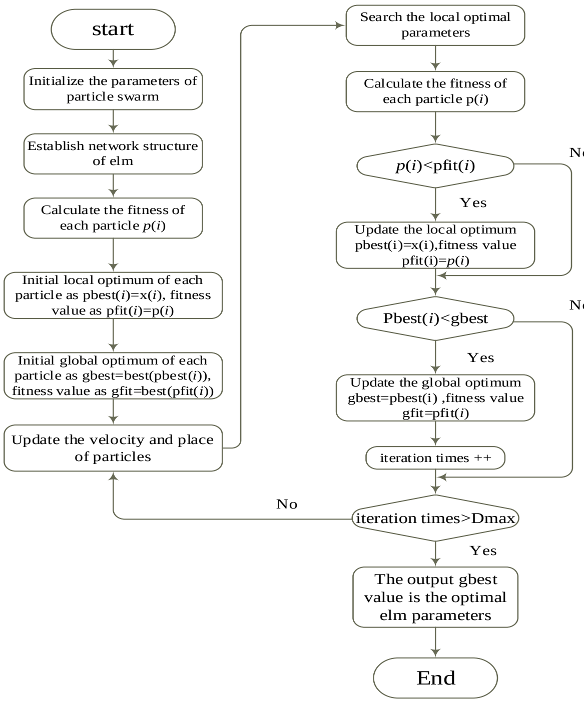

Fig.3. Spline-OS-ELM algorithm description for ELM parameters training

IV. RESULTS AND DISCUSSIONS

According to the Lipschitz exponent[12], two different types of singularities will be introduced into the corresponding original data, which will be trained and predicted in this section.

Firstly, a typical exponential degradation-signal model in order to further test this approach presented in this paper. This exponential degradation model is a stochastic process which captures the evolution of the degradation signal. The model has the form as follows:

$$Y(t) = \varphi + \theta \exp\left\{\beta t + \varepsilon(t) - \frac{\sigma^2}{2}\right\} \qquad (14)$$

where $Y(t)$ denotes the degradation signal as a continuous stochastic process, continuous with respect to time t, φ is a constant deterministic parameter, θ is a lognormal random variable, where $ln\theta$ has mean μ0 and variance σ_0^2, β is a normal random variable with mean μ1 and variance σ_1^2, and $\varepsilon(t)$ is a normally distributed random error term with mean 0 and variance σ^2. Let the model parameter φ=0.12, μ0 =0.05, σ_0^2=2.5×10-7, μ1 =1.1, σ_1^2=1×10-8, and σ^2=1×10-6. Two singular points are set at point 245 and point 318 as shown in figure 4.

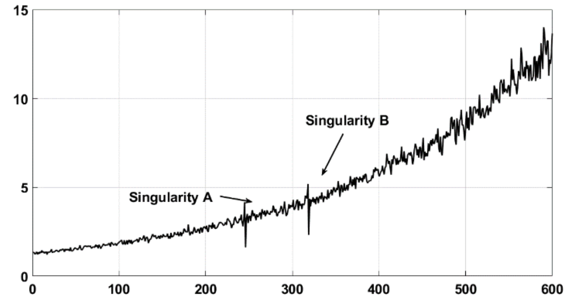

Fig.4. The simulation data of the exponential degradation-signal model

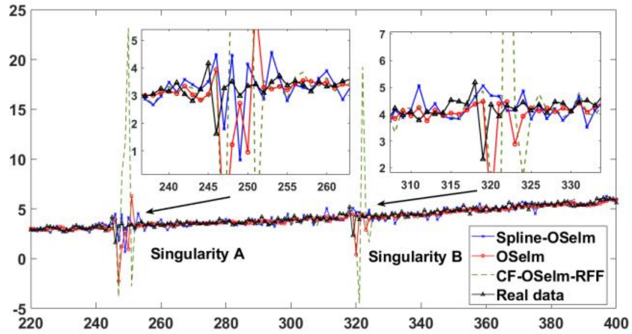

Fig.5. The prediction results based on different models

Figure 5 shows the prediction results based on three different methods. As seen from this figure, Spline-OS-ELM model (the bulue line) has a better prediction performance than OS-ELM and Cholesky Factorization Based Online Sequential Extreme Learning Machines with Persistent Regularization and Forgetting Factor (CF-OS-ELM-PRFF) method[13]. This proposed approach could track the actual curve very rapidly after the singular point. The prediction errors are listed in Table 1 as follows. Clearly, the influence of singularity on forecasting performance of Spline-OS-ELM model is less than that of other 2 models.

Then, a typical time-series dataset is introdued to test this approach presented in this paper. Another types of two singularity points were set into this Mackey-Glass (MG) simulation dataset in figure 6. Figure7 shows the singularity points via the detail signal of wavelet decomposition.

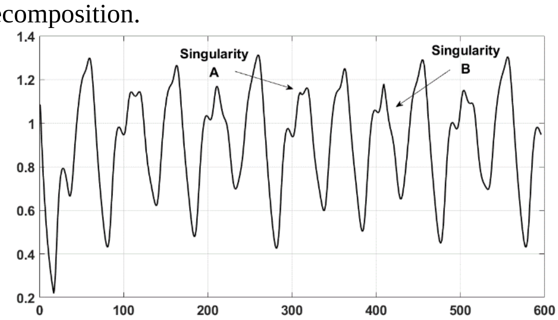

Fig.6. The Mackey-Glass simulation dataset

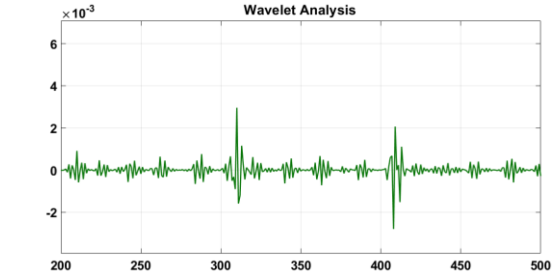

Fig.7. Wavelet singularity detection for Mackey-Glass simulation dataset

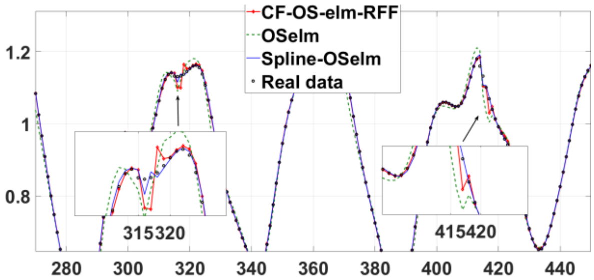

Fig.8. The prediction results based on different models

The prediction results is in figure 8. To the hidden and latent fault points, Spline-OS-ELM model (the blue line) also shows a better prediction performance, the error comparison is listed in Table 1 as well.

In order to keep on evaluating the predictive ability of the proposed method in practical engineering under the effect of singularity, a degradation data set from a radio frequency (RF) low-noise amplifier circuit (shown in figure 9) is analyzed here. The RF circuit is from a typical receiver channel module with the input range of 1mVpp~10Vpp, bandwidth of DC~200MHz, and the output signal of 500mVpp. We take its signal gain as the monitored parameter in the degradation trend analysis, due to the gain is the most sensitive parameter among these parameters.

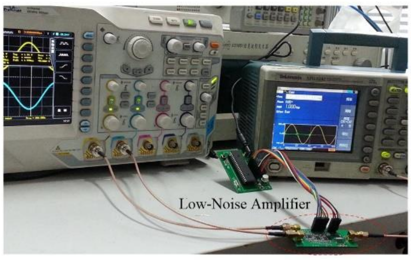

Fig. 9. A radio frequency (RF) low-noise amplifier circuit

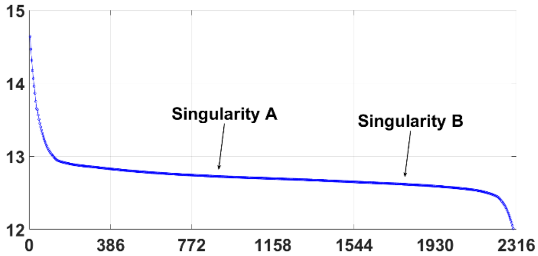

Fig.10. The low-noise amplifier aging data

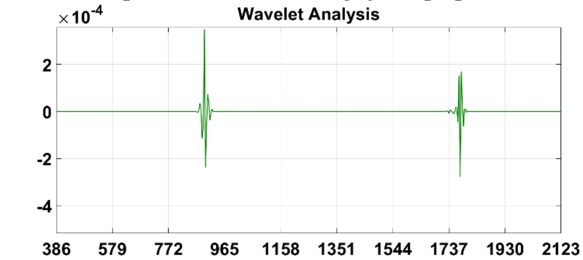

Fig.11. Wavelet singularity detection for the aging data of low-noise amplifier

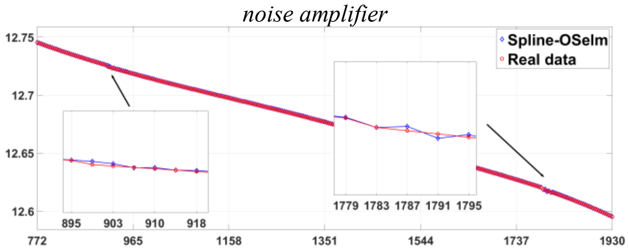

Fig.12. The forecasting of spline-OS-ELM model for low-noise amplifier aging data

A total of 600 points were collected from the low-noise amplifier under 100°C, this sequence lasted 2316 hours and contained two singular points at around 899 hours and 1783 hours respectively which are extremely unobvious (shown in figure 10). A db5 wavelet decomposition of 4 layers was applied to the data in order to observe it in figure11.

The one-step forecasting result of spline-OS-ELM model for low-noise amplifier aging data (real data: red circle line, and forecasting results: blue diamond line) is shown in figure 12. As seen from Fig.7, although there exist two singular points, the forecasting results are still fit perfectly the actual data.

To further evaluate the prediction ability, a comparison among different models is performed on the same aging data, and the prediction errors of these models are shown in Table 1. The three-dimensional histogram of the root-mean-square deviation or error (RMSE) is as shown in Figure 13. From the comparison results, although test sequences contain two singular points, the proposed method is still efficient and do not show any deterioration, the prediction error of the proposed method is lower than the other two methods.

Table 1. Prediction accuracy of different models

Models		Exponential dataset	MG dataset	RF Circuit

OS-ELM	RMSE	4.210e-002	4.33e-002	2.326e-002
CF-OS-ELM-RFF	RMSE	1.183e-001	6.908e-003	3.901e-003
Spline-OS-ELM	RMSE	1.125e-002	1.714e-003	1.416e-003

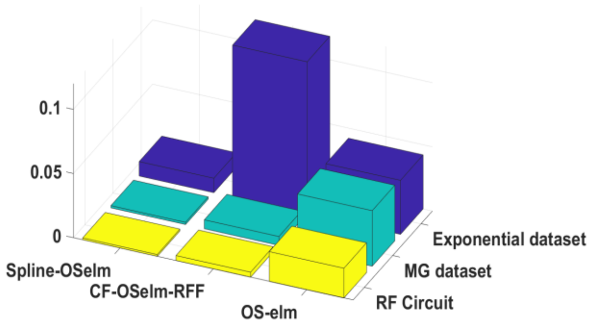

Fig. 13. The three-dimensional histogram of the RMSE of different prognostic models

According to the theoretical and practical experimental results of the presented algorithm, it can be applied well in many kinds of fault situations, especially the possible intermittent fault and the singular point fault.In this way, the singularity is not eliminated as noise, but kept as an effective information in historical record.

V. CONCLUSIONS AND OUTLOOK

Singularity perturbation existed in performance degradation data trend, sometimes it can cause large effects, and obviously reduce forecasting accuracy. In order to tackle this perplexing issue, in this paper, a novel ensemble prognostic approach is proposed.

As a surrogate model of the signal trend analysis, the cubic non-parameter spline shows its flexibility and wide applicability. The particle swarm optimization (PSO) method is combined to improve the OS-ELM algorithm, The This improved method finally determinates the optimal spline cell and outputs the optimal spline parameters.

The major advantage of the proposed method is that it does not need to identify the singularity in a time series, and this is a very important in some cases of no effective method to identify singular points. Comparing with other two OS-ELM methods, this method does not only improve the prediction accuracy, but also could provide more comprehensive information records for decision makers to ensure the effective monitoring for potential risks.

VI. ACKNOWLEDGMENTS

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REFERENCES

[1] **Mittal, R. C., Sumita Dahiya.** : A comparative study of modified cubic B-spline differential quadrature methods for a class of nonlinear viscous wave equations. Engineering Computations 35.1, 2018, pp. 315-333.

[2] **G. Huang, N. Liang, H. Rong, P.Saratchandran, and N. Sundararajan.** On-line sequential extreme learning machine, in Computational Intelligence, 2005,pp. 232-237.

[3] **Tang J , Deng C , Huang G B .** : Extreme Learning Machine for Multilayer Perceptron[J]. IEEE Transactions on Neural Networks & Learning Systems, 2017:809-821.

[4] **Liang N Y, Huang G B.** A Fast and Accurate Online Sequential Learning Algorithm for Feedforward Networks[J].*IEEE Transactions on Neural Networks*, 2006,17(6):1411-1423.

[5] **Lan Y,Soh Y C, Huang G.** Ensemble of online sequential extreme learning machine [J].*Neurocomputing*, 2009, 72(13-15):3391-3395.

[6] **Guo L, Hao J, Liu M.** An incremental extreme learning machine for online sequential learning problems [J]. *Neurocomputing*, 2014,128(27):50-58.

[7] **Shao Z, Er M J.** An online sequential learning algorithm for regularized Extreme Learning Machine[J]. Neurocomputing, 2016: 778-788.

[8] **Sun J, Wu X, Palade V, et al.** Convergence analysis and improvements of quantum-behaved particle swarm optimization[J]. Information Sciences, 2012: 81-103.

[9] **J. Rashidinia and Z. Mahmoodi.** Non-Polynomial Spline Solution of a Singularly-Perturbed Boundary-Value Problems, *Int. J. Contemp. Math. Sciences*, vol. 2, pp. 1581-1586, 2007.

[10] **A. Khan, M. A. Noor, and T. Aziz.** Parametric quintic-spline approach to the solution of a system of fourth-order boundary-value problems, *Journal of optimization theory and applications*, vol. 122, pp. 309-322, 2004.

[11] **S. S. Siddiqi and G. Akram.** Solution of the system of fourth-order boundary value problems using non-polynomial spline technique, *Applied Mathematics and Computation*, vol. 185, pp. 128-135, 2007.

[12] **S. Mallat and W. L. Hwang.** Singularity detection and processing with wavelets, *IEEE transactions on information theory*, vol. 38, pp. 617-643, 1992

[13] **Zhou X , Kui X .** Cholesky Factorization Based Online Sequential Extreme Learning Machines with Persistent Regularization and Forgetting Factor[J]. Symmetry, 2019, 11(6):801.