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A consensus ranking based proposal for combining data in adjustment of the fundamental physical constant values

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***Abstract* – To ensure the traceability of testing and diagnostic equipment it is necessary to provide a chain of comparisons connecting the equipment with primary standards of the SI units based on fundamental physical constants. The values of the constants are regularly determined by an adjustment procedure which requires consistency of input data and an assumed statistical model. In this paper, it is proposed to apply the developed interval fusion with preference aggregation (IF&PA) method for combining data and determining a consensus value of a fundamental constant. Due to its high robustness, accuracy and reliability confirmed by the numerical experimental results, the IF&PA does not require a consistency check and works well without using any statistical assumptions. Usage of the IF&PA is demonstrated by example of processing the simulated interval data and real values of the Planck constant used in the adjustment in 2006 and 2017. The outcome comparison with the estimates obtained by other methods, including the procedures based on Birge ratio, modified Birge ratio, random effects model and fixed effects model, is carried out.**

Keywords* – *Fundamental Constant Adjustment, Consensus Estimate, Ranking, Data Fusion, Preference Aggregation.

I. INTRODUCTION

According to international requirements [1], all measuring instruments (including those intended for testing and diagnostics) have to be calibrated before putting into operation; they also should be recalibrated within prescribed calibration period. The programme for calibration is supposed to ensure that measurement results are traceable to the SI units. Traceability is established by means of an unbroken chain of comparisons that connects measuring instruments with primary standards, which are agreed representations of the SI units based on fundamental physical constants. Thus, the accuracy of any measuring, testing or diagnostic procedure directly depends on the correctness of determination of the constant values.

A self-consistent set of internationally recommended values of the basic fundamental constants is periodically provided by the Committee on Data for Science and Technology (CODATA) [2]. These values of the constants along with their quoted uncertainties are determined by a special procedure called *adjustment*. The adjustment is the processing of numerical set of measured (or calculated) values, obtained from experiment or theory by different methods and provided from different laboratories, to estimate the best *consensus value* of a constant. Every four years, the CODATA Task Group on Fundamental Constants carries out a *least squares adjustment* (LSA) of the fundamental constants taking into account all relevant data from diverse researches [3].

The obligatory condition for the adjustment procedure is a *consistency* of input data. It is an often situation where the data provided by different laboratories are inconsistent, i.e. discrepancy between the input values are larger than it is expected taking into consideration the quoted uncertainties. Such inconsistency is caused by some unknown sources of uncertainty, which should be identified, but in practice it is hard to implement. To deal with data inconsistencies, the Task Group has to omit some values from the adjustment set or increase the initial uncertainties of the values by an expansion factor.

More to the point, the determination of a consensus value of a constant on the basis of an inconsistent input data set requires additional assumptions. Thus, existing techniques used for the adjustment, including LSA, suppose that the input data are normally distributed, independent random variables with the same mean μ and exactly known variances u_k^2, but this hypothesis is not always valid [4].

This paper is the authors' first attempt to apply an alternative approach for combining the data and determining a consensus value of a fundamental constant. The approach is based on our interval fusion with preference aggregation (IF&PA) method [5], which is found to be highly robust to successfully deal with inconsistent data. The IF&PA doesn't require any consistency check as well as any assumptions about the distribution law and independency of data, and uses all the available values of a constant for the adjustment procedure.

II. COMBINING DATA IN ADJUSTMENT OF THE FUNDUMENTAL CONSTANTS

In this Section, the procedure for combining data used in adjustment of the fundamental constants is considered.

Let $x_1, \ldots, x_m$ be a set of measured (or calculated) values of the same fundamental constant, along with corresponding standard uncertainties $u_1, \ldots, u_m$. The aim is to combine these data for obtaining a single estimate x^*, and to determine the uncertainty u^* associated with x^*.

The estimate x^* is typically calculated as a weighted mean

$$x^* = \sum_{k=1}^{m} u_k^{-2} x_k \,/ \sum_{k=1}^{m} u_k^{-2} \qquad (1)$$

with corresponding uncertainty

$$u^* = \left(1 / \sum_{k=1}^{m} u_k^{-2}\right)^{1/2}, \qquad (2)$$

where m is a number of values of a constant.

Decision about the consistency of the data set is made by conducting the chi-square test and computing the statistics:

$$\chi^2 = \sum_{k=1}^{m} (x^* - x_k)^2 / u_k^2 , \qquad (3)$$

where the χ^2 is a value of a chi-square random variable with $v = m - 1$ degrees of freedom.

Among procedures used for combining inconsistent data one of the most popular is a procedure based on the Birge ratio [6]. It is applied by the CODATA Task Group for the adjustment. The procedure assumes that values x_k are normally distributed with mean μ and variance $c^2 u_k^2$, $k = 1, \ldots, m$. Then the standard uncertainty of the x^*, is expanded and estimated as $u_B^* = cu^*$.

To evaluate c, the Birge ratio, which is a measure of the data consistency, is calculated with v degrees of freedom:

$$R_B = \sqrt{\chi^2 / v} = c . \qquad (4)$$

In case of combining data of the same constant, the value (χ^2 / m – 1) is an unbiased estimator of c^2, hence, the standard uncertainty of x^* is computed as follows:

$$u_B^* = u^* \sqrt{\chi^2 / m - 1} . \qquad (5)$$

In literature, there are alternative procedures for combining data in the adjustment of the constants, for example, procedures based on the random effects model [7, 8] or fixed effects model [7, 9]. This paper is not focused on their detailed description, but exploits the results obtained by these procedures in [7] for comparison with our IF&PA method.

III. INTERVAL FUSION WITH PREFERENCE AGGREGATION

This Section presents the interval fusion with preference aggregation (IF&PA) method for combining inconsistent data in adjustment of the fundamental constants.

Every value x_k of a fundamental constant along with its uncertainty u_k can be represented as an *interval* I_k on the real line. We consider a set of initial intervals I_k, $k = 1, \ldots, m$, as input data for the IF&PA method.

By *interval data fusion* we understand a procedure of determination of a resulting interval [$x^* \pm u^*$] which is consistent with the maximal number of initial intervals I_k, $k = 1, \ldots, m$, and with maximum likelihood contains a point x^* (fusion result) that can be considered as a representative of all I_k with an uncertainty u^*.

The fusion result x^* is supposed to be selected from finite set of points related to initial intervals. To form this finite set we introduce a concept of a range of actual values (RAV) $A = \{a_1, a_2, \ldots, a_n\}$ consisting of ordered discrete values a_i, $i = 1, \ldots, n$.

The RAV is composed by union of all intervals I_k with the lower bound $a_1 = \min\{l_k \mid k = 1, ..., m\}$ and the upper bound $a_n = \max\{u_k \mid k = 1, ..., m\}$, where l_k and u_k are the lower and upper bounds of some k-th interval correspondently. Next, we partition this union into n – 1 equal subintervals of length h to obtain elements $a_2, a_3, \ldots, a_{n-1}$. Number n, or cardinality of the RAV partition, is calculated by (6):

$$n = \lceil (a_n - a_1) / 0.31\sigma \rceil + 1 , \qquad (6)$$

where σ is a standard deviation value computed before RAV partition [5]. After partitioning, the length h of interval is defined as $h = |a_i - a_{i-1}|$, $i = 2, \ldots, n$. Thus, as a result of RAV partitioning it is obtained a set $A = \{a_1 < a_2 < \ldots < a_i < \ldots < a_n\}$ of fully ordered discrete values a_i, $i = 1, \ldots, n$, which are then ranked in a specific way to introduce every particular interval.

On the next stage, the initial intervals I_k are represented by rankings $\lambda_k = (a_1 \succ a_2 \succ \ldots \sim a_n)$ over a set of n discrete values $A = \{a_1, a_2, \ldots, a_n\}$ *belonging to* these intervals. Ranking λ_k induced by interval I_k, or *inranking*, is composed according to conditions (7) for $i, j = 1, \ldots, n$:

$$\begin{array}{l} \text{(i) } a_i \in I_k \wedge a_j \notin I_k \Rightarrow a_i \succ a_j; \\ \text{(ii) } a_i,a_j \in I_k \vee a_i,a_j \notin I_k \Rightarrow a_i \sim a_j; \\ \text{(iii) } a_i \notin I_k \wedge a_j \in I_k \Rightarrow a_i \prec a_j; \\ \text{(iv) } a_i,a_j \in I_k \text{ are neighbors } \Rightarrow j \equiv i + 1 \end{array} \qquad (7)$$

A set of m inrankings $\Lambda = \{\lambda_1, \lambda_2, \ldots, \lambda_m\}$ called *preference profile* is formed for initial intervals I_k. For Λ we determine a single preference relation (or *consensus ranking*) β which represents the best compromise between the initial inrankings. To do this, we apply recursive branch and bounds algorithm implementing the Kemeny rule [10]. It finds such a linear order β of the alternatives of A that the distance $D(\beta, \Lambda)$ (defined in terms of the number of pairwise inconsistencies between inrankings) from β to the inrankings of the initial profile $\Lambda(m, n)$ is minimal for all possible strict orders ρ, that is

$$\beta = \arg\min \sum_{i<j} p_{ij} , \quad (8)$$

where $p_{ij} = \sum_{k=1}^{m} [1 - \mathrm{sgn}(a_i^k, a_j^k)]$ is an element of the $(n\times n)$ profile matrix $[p_{ij}] = P$, rows and columns of which are labeled by the alternatives' numbers; $\mathrm{sgn}(a_i, a_j)$ is a function that reveals the sign (or direction) of the pair $(a_i,a_j)\in\lambda$. The recursive algorithm determines multiple consensus rankings $\beta_1, \beta_2, \ldots, \beta_N$ which are transformed into a single final consensus ranking β_{fin} by means of a convolution rule [11].

The convolution rule is formulated as follows. Let $\mathrm{B}(N, n) = \{\beta_1, \beta_2, \ldots, \beta_N\}$, $\mathrm{B} \subset \Pi_n$, be a set of consensus rankings for a profile $\Lambda(m, n) = \{\lambda_1, \lambda_2, ..., \lambda_m\}$ given over some set of alternatives $A = \{a_1, a_2, ..., a_n\}$. Let *rank* r_i^t be a position of an alternative a_i in the consensus ranking $\beta_t \in \mathrm{B}$, $t = 1, \ldots, N$. Then, for all $i < j$, $i, j = 1, \ldots, n$, we have

$$\sum_{t=1}^{N} r_i^t < \sum_{t=1}^{N} r_j^t \Rightarrow a_i \succ a_j \text{ , and } \sum_{t=1}^{N} r_i^t = \sum_{t=1}^{N} r_j^t \Rightarrow a_i \sim a_j, \quad (9)$$

where both strict order relation $\succ$ and tolerance relation $\sim$ are in the single final consensus ranking β_{fin}.

The main stages of the IF&PA method are as follows.

1. Forming the RAV from initial intervals I_k, $k = 1, \ldots, m$ and partitioning it into $n - 1$ subintervals in accordance with (6) to obtain a set $A = \{a_1, a_2, ..., a_n\}$.
2. Composing preference profile $\Lambda = \{\lambda_1, \lambda_2, ..., \lambda_m\}$ consisting from inrankings in accordance with (7).
3. Determining a fusion result x^* as the best alternative in consensus ranking β_{fin} by (8-9).
4. Determining an uncertainty u^* of the fusion result x^* by the formula (10)

$$u^* = \min(\max_{k=1,\ldots,m'} \{l_k \le x^*\}, \min_{k=1,\ldots,m'} \{u_k \ge x^*\}) , \quad (10)$$

where m' is a number of intervals from I_k which include the value x^*.

IV. RESULTS AND DISCUSSIONS

This Section presents the results of processing of interval data by the IF&PA as well as the comparison with the results obtained by other methods. The processed data are both simulated random values and real Planck constant values from the CODATA set.

A. Processing data of numerical simulation

To verify the proposed IF&PA method for combining the data we developed special software IntFusion in the NI LabVIEW environment. The software generates synthetic pseudo-random interval data, which represent measured values of the Planck constant with relative uncertainties as the pairs $<x_k, u_k>$, $k = 1, \ldots, m$, using Monte-Carlo method [12].

In experiments, the CODATA recommended value 6.626 070 150(69) × 10^{-34} J s was used as a nominal (true) value x_{nom} of the measured quantity. The number m of intervals was set to be 9, and there were conducted 100 individual experiments for both normal and uniform distributions of interval data.

Since the nominal value is preset in simulation one can judge the procedure performance by the deviation ξ of the obtained value x^* from the nominal value x_{nom}:

$$\xi = |x_{\mathrm{nom}} - x^*|. \quad (11)$$

The deviation ξ allows to estimate a set of three characteristics of the procedure: robustness, accuracy, and reliability. We treat *robustness* as an independence of the procedure outcome x^* on a particular law of interval data distribution. Under *accuracy* we understand a closeness of the outcome x^* to the nominal value x_{nom}. *Reliability* is considered as a degree of belief to the obtained outcomes x^*.

The deviation ξ can be graphically demonstrated for the visual comparison of robustness, accuracy, and reliability of the procedures. The outcomes x^* of 100 experiments is presented as a curve $\xi(v)$, where values ξ are sorted in ascending order and v is a number of a particular experiment. On the graph, robustness is demonstrated by the discrepancy between the curves $\xi(v)$ for normal and uniform distributions; the smaller the discrepancy the higher the robustness of a procedure. Accuracy is determined by the distance between the curve $\xi(v)$ and x-axis.

As a measure of reliability the probabilities $P(\xi \le \xi_b)$ were used that the value ξ does not exceed some fixed *boundary deviation* ξ_b. For instance, if $\xi \le 0.5$ in 95 experiments out of the 100, then the probability $P(\xi \le 0.5) = 0.95$.

The generated interval data were processed by the procedure IF&PA and by the procedure based on Birge ratio. The resulting estimate x^*, its relative uncertainty u_r^* = u^*/x^* and deviations ξ were determined. An example of the simulated values of x_k (which represent Planck constant values h_k) and $u_r = u_k / x_k$ for one arbitrary selected experiment are presented in Table 1.

Table 1. Simulated measured values of the Planck constant with corresponding relative uncertainties.

Number k of interval	$x_k = h_k$, J s	u_r
1	6.626 070 138 × 10^{-34}	3.2×10^{-9}
2	6.626 070 163 × 10^{-34}	4.1×10^{-9}
3	6.626 070 152 × 10^{-34}	3.3×10^{-10}
4	6.626 070 144 × 10^{-34}	3.0×10^{-10}
5	6.626 070 144 × 10^{-34}	1.0×10^{-9}
6	6.626 070 155 × 10^{-34}	3.3×10^{-9}
7	6.626 070 143 ×10^{-34}	4.6×10^{-9}
8	6.626 070 157 ×10^{-34}	1.1×10^{-9}
9	6.626 070 174 ×10^{-34}	1.4×10^{-9}

After the processing, for the particular example the following results were obtained:

- for the IF&PA method – the resulting estimate $x^* = 6.626\ 070\ 152 \times 10^{-34}$ J s, the relative standard uncertainty $u_r^* = 2.34 \times 10^{-10}$, the deviation $\xi = 8.58 \times 10^{-10}$ J s;
- for the Birge ratio procedure – the resulting estimate $x^* = 6.626\ 070\ 148 \times 10^{-34}$ J s, the relative standard uncertainty $u_r^* = 2.09 \times 10^{-10}$, the deviation $\xi = 2.30 \times 10^{-9}$ J s.

The simulated interval data as well as the obtained outcomes of two procedures are shown in Fig 1. The dashed line on the graph indicates the nominal value.

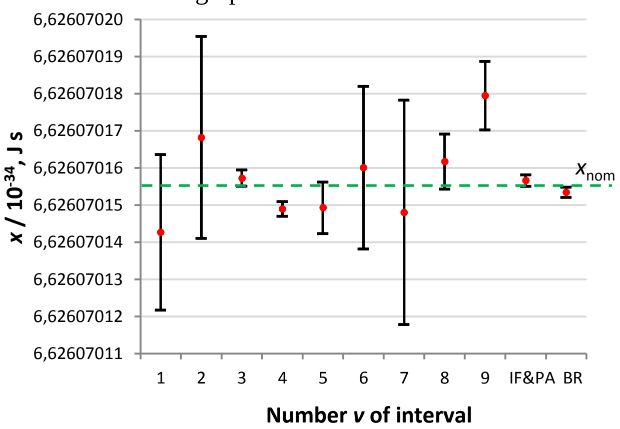

Fig. 1. An example of simulated interval data and the outcomes of the IF&PA and the Birge ratio procedures.

For the given example, the relative uncertainties for both procedures are almost the same, while resulting estimate obtained by the IF&PA is closer to the nominal value.

To evaluate the characteristics of procedure performance, the curves $\xi(v)$ were constructed (Fig. 2). One can see from Fig. 2 that the average discrepancy between normal and uniform distributions (curves 3 and 4), in case of the IF&PA, is about 0.60×10^{-7}, whereas for the Birge ratio procedure (curves 1 and 2) the discrepancy is three times larger – about 1.99×10^{-7}. These results denote the noticeably higher robustness of the IF&PA against the Birge ratio procedure.

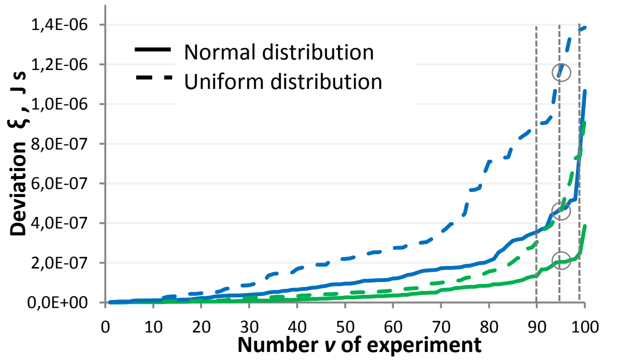

Fig. 2. Deviations ξ obtained by the IF&PA and the Birge ratio procedures for uniform and normal distributions of data.

It is evident from Fig. 2 that the curves $\xi(v)$ for the IF&PA are closer to the x-axis than those obtained by the Birge ratio procedure for both normal and uniform distribution. The average distances to the x-axis are 0.52×10^{-7} and 1.48×10^{-7} (case of normal distribution), and 1.12×10^{-7} and 3.47×10^{-7} (case of uniform distribution), for the IF&PA and the Birge ratio procedure correspondently. Therefore, the IF&PA procedure have demonstrated higher (approximately 3 times) accuracy than the Birge ratio procedure.

The calculated values of boundary deviations ξ_b for probabilities $P = 0.90$, $P = 0.95$ and $P = 0.99$ are shown in Table 2. They are also illustrated in Fig. 2 as cross points of the curves $\xi(v)$ with dashed perpendiculars to the x-axis.

Table 2. Boundary deviations ξ_b for probabilities P = 0.90, P = 0.95 and P = 0.99.

Probability	Boundary deviation ξ_b, 10^{-7}·J s			
	IF&PA		Birge ratio	
	Normal	Uniform	Normal	Uniform
0.90	1.33	3.04	3.54	8.80
0.95	2.04	4.50	4.73	11.77
0.99	2.50	7.42	7.73	13.73

It follows from the data given in Table 2 that for all considered probabilities P the smallest values of boundary deviations ξ_b were provided by the IF&PA procedure.

Thus, the IF&PA procedure has shown improved performance characteristics, namely robustness, accuracy, and reliability, in comparison with the Birge ratio procedure.

Fig. 3 demonstrates the obtained values of the the relative standard uncertainty u_r^* sorted in ascending order for 100 numerical experiments. As can be seen, the obtained uncertainties are of the same order of magnitude and in general commensurate for two procedures considered in the experiment.

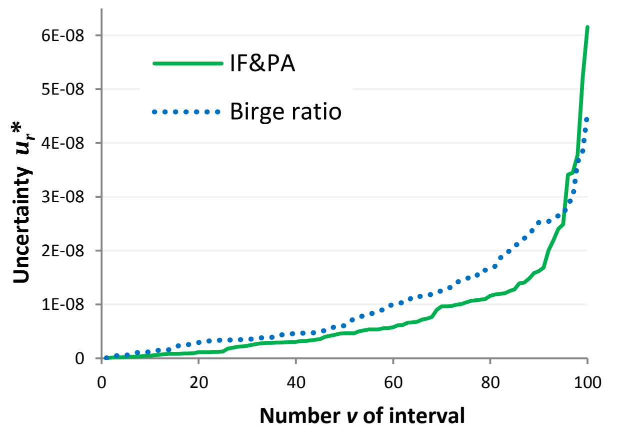

Fig. 3. Relative uncertainties u_r obtained by the IF&PA and the Birge ratio procedures.

B. Processing data of the CODATA adjustment 2006

For the sake of comparison simplicity, we analyzed data set from [7], which represents the values of the Planck constant *h* (Table 3). The set includes eight values used for the CODATA adjustment of 2006, and four values obtained after that adjustment. Twelve measured values of the Planck constant *h* with corresponding relative uncertainties u_r are also shown in Fig.4.
The resulting estimate of the Planck constant obtained by the IF&PA is given in Table 4 and graphically demonstrated in Fig.4.

Table 3. Measured values of the Planck constant with relative uncertainties as in [7].

Identification	*h*, J s	u_r
NPL-79	6.626 0729 × 10^{-34}	1.0×10^{-6}
NIST-80	6.626 0657 × 10^{-34}	1.3×10^{-6}
NMI-89	6.626 0684 × 10^{-34}	5.4×10^{-7}
NPL-90	6.626 0682 × 10^{-34}	2.0×10^{-7}
PTB-91	6.626 0670 × 10^{-34}	6.3×10^{-7}
NIM-95	6.626 071× 10^{-34}	1.6×10^{-6}
NIST-98	6.626 068 91 ×10^{-34}	8.7×10^{-8}
NIST-07	6.626 068 91 ×10^{-34}	3.6×10^{-8}
METAS-11	6.626 0691 ×10^{-34}	2.9×10^{-7}
NPL-12	6.626 0712 ×10^{-34}	2.0×10^{-7}
NRC-12	6.626 070 63 ×10^{-34}	6.5×10^{-8}
Avogadro-11	6.626 070 09 ×10^{-34}	3.0×10^{-8}

In [7], the following combining data procedures were considered based on: (1) Birge ratio, (2) modified Birge ratio, (3) random effects model and (4) fixed effects model. The outcomes obtained by means of these procedures are presented in Table 4.

Table 4. Resulting estimates of the Planck constant for data from Table 3.

Method	*h**, J s	u_r^*
Birge ratio	6.626 069 67 × 10^{-34}	3.13 × 10^{-8}
Modified Birge ratio	6.626 069 67 × 10^{-34}	3.46 × 10^{-8}
Random effects	6.626 069 60 × 10^{-34}	6.68 × 10^{-8}
Fixed effects	6.626 069 60 × 10^{-34}	9.21 × 10^{-8}
IF&PA	6.626 069 34 × 10^{-34}	1.43 × 10^{-8}

As can be seen, the resulting estimate *h** obtained by the IF&PA noticeably differs from the others. It is expectable since the procedures (1)–(4) estimate the value *h** by calculating a weighted mean using formula (1) directly or with some modifications. This yields almost similar values of the estimate for all procedures (1)–(4).
On the other hand, Fig.4 demonstrates that the IF&PA provides the best compromise value for the largest subset of consistent intervals without preliminary elimination of any initial interval. Moreover, the IF&PA has allowed to have the resulting estimate of the Planck constant with a lower standard uncertainty u_r^*.

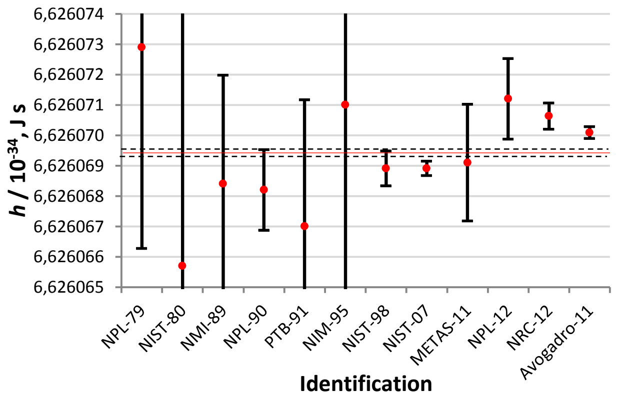

Fig. 4. Measured data of the Planck constant as in [7]. The IF&PA resulting estimate is demonstrated by the red line with the uncertainty boundaries denoted by dashed lines

C. Processing data of the CODATA special adjustment

The IF&PA method was applied to processing the latest values of the Planck constant *h* (Table 5) used for the CODATA special adjustment in 2017 carried out for revision of International System of Units [13, 14].

Table 5. Measured values of the Planck constant with relative uncertainties as in [13].

Identification	*h*, J s	u_r
NRC-17	6.626 070 133(60) × 10^{-34}	9.1×10^{-9}
IAC-17	6.626 070 405(77) × 10^{-34}	1.2 ×10^{-8}
NIST-17	6.626 069 934(88) × 10^{-34}	1.3×10^{-8}
IAC-15	6.626 070 22(13) × 10^{-34}	2.0×10^{-8}
NMIJ-17	6.626 070 13(16) × 10^{-34}	2.4×10^{-8}
IAC-11	6.626 069 94(20) × 10^{-34}	3.0×10^{-8}
LNE-17	6.626 070 40(38) × 10^{-34}	5.7×10^{-8}
NIST-15	6.626 069 36(38) × 10^{-34}	5.7×10^{-8}
NIST-98	6.626 068 91(58) × 10^{-34}	8.7×10^{-8}

The results of processing data from Table 5 by the IF&PA are illustrated in Fig.5. The resulting estimates *h** of the Planck constant along with the relative standard uncertainties u_r^* obtained by the IF&PA and by the Birge ratio procedure (i.e. CODATA 2017 recommended value) are shown in Table 6.

Table 6. Resulting estimates of the Planck constant for data from Table 5.

Method	*h**, J s	u_r^*
Birge ratio	6.626 070 150 (69) × 10^{-34}	1.0 × 10^{-8}
IF&PA	6.626 070 170 (90) × 10^{-34}	3.5 × 10^{-9}

One can see that the IF&PA estimate is quite close to the one recommended in [13], but has a much lower uncertainty. It should be mentioned that the IF&PA exploited all nine available values of the Planck constant to find a result, while the Birge ratio procedure did not include the value of NIST-98 in the adjustment.

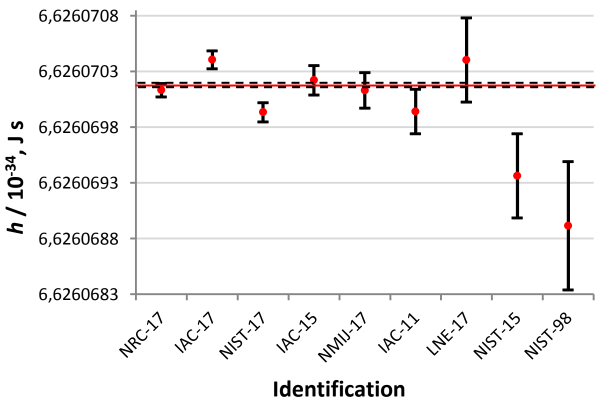

Fig. 5. Measured data of the Planck constant as in [13]. The IF&PA resulting estimate is demonstrated by the red line with the uncertainty boundaries denoted by dashed lines

V. CONCLUSIONS AND OUTLOOK

We proposed an alternative approach for combining inconsistent data in adjustment of the fundamental constants which we called the interval fusion with preference aggregation (IF&PA) method. In contrast with other procedures, it does not need a preliminary consistency check. As demonstrated by the results of numerical experimental investigations, the IF&PA shows high robustness, accuracy, reliability, and works well without using any statistical assumptions.

The results of processing real values of the Planck constant have shown that the resulting estimates provided by the IF&PA method differ from estimates obtained by Birge ratio procedure, modified Birge ratio procedure, procedures based on the random effects model and on the fixed effects model. This is caused by the fact that the IF&PA does not assign weights to the data or eliminate inconsistent values, i.e. it combines all available input data. However, we may consider the outcomes of the IF&PA method as rather reliable, as the experimental investigation results demonstrated the ability of the IF&PA to provide accurate (close to the true value) resulting estimate.

In case of processing real data, the standard uncertainty determined by the IF&PA is lower in comparison with other procedures. But we should note that the IF&PA method supposes to find the "best case" value of uncertainty. In practice it could be too optimistic, and to overcome this difficulty the dependence of the uncertainty on the RAV partition should be further investigated. Another line of development of the IF&PA concerns taking into account possible correlation among data.

VI. ACKNOWLEDGMENTS

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