

# Experimental strain modal analysis for beam-like structure by using distributed fiber optics

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**Abstract – Modal analysis is commonly considered as an effective tool to obtain the inherent characteristics of structures including natural frequency, modal damping and mode shapes, which are significant indicators for monitoring the health status of engineering structures. In this paper, distributed fiber optics, as dense measurement transducer has been applied into acquiring huge amount of strain data along the beam surface. Thanks to the dense spatial resolution, CMIF (Complex Mode Indicator Function) is used to identify strain modal parameters like natural frequency and modal damping. Strain mode shapes can be obtained by using SVD technique.**

## I. INTRODUCTION

Strain is a significant indicator that reflects the internal health status of engineering structures. Many infrastructures are inevitable to suffer from unpredictable external vibration source, fatigue as one of common damage might be generated, which is strongly related to the strain and stress status of structures. In order to make clear the dynamic characteristics of the structure, strain modal analysis technique should be applied into practical engineering structures [1-3]. Considering the usage of transducer for acquiring vibration response from the structure, traditional transducers reflect some unavoidable shortcomings, such as the influence that comes from weight of the transducer, the limited number of transducer for acquisition configuration, and so on.

Based on the shortcomings of traditional transducers, distributed fiber optics, as one of dense strain transducers, has attracted lots of attention from researchers [4-6], considering that it can acquire huge amount of strain data in the same time and makes little effect on the response in terms of weight. Strain modal analysis is gaining attention as structural health monitoring tool and it could be rather useful for fiber optic transducers. method will be introduced into strain modal analysis by using distributed fiber optics. CMIF method [7-8], as a common modal analysis approach, has been used rarely into strain modal analysis, the relationship among displacement modal analysis, strain modal analysis and CMIF method has been investigated.

In this article, the content is arranged as following: In section 1, strain modal parameter technique is presented. Simulation validation on a clamped-clamped beam has

been studied and presented in the section 2. In section 3, distributed fiber optics has been applied into a clamped-clamped steel beam, strain modal analysis has been performed and shows its feasibility and effectiveness. CMIF

## STRAIN MODAL PARAMETERS IDENTIFICATION

### 1. CMIF (Complex Mode indicator function)

Complex mode indicator function has been proposed for several decades and it is mainly used to find out the repeated roots of traditional FRF matrix in order to obtain the appropriate modal parameters, especially when natural frequencies are close [9-10]. When combining with enhanced frequency response function (eFRF), namely enhancing the particular vibration mode respectively [11], the definition of eFRF is based on the transformation from physical domain into modal domain. Therefore CMIF is competent to perform modal parameters identification with comparison to other modal analysis techniques. The traditional frequency response function (FRF) can be defined as

$$H_{pq}(\omega) = \sum_{r=1}^{2N} \frac{Q_r \psi_{pr} \psi_{qr}}{j\omega - \lambda_r} \quad (1)$$

where:

$Q_r$  is scaling factor for  $r$ th mode

$\psi_{pr}$  is the  $r$ th mode shape for output DOF  $p$

$\psi_{qr}$  is the  $r$ th mode shape for input DOF  $q$

$\lambda_r$  is  $r$ th system pole value

$H_{pq}$  can be redefined as:

$$H_{pq}(\omega) = \sum_{r=1}^{2N} \psi_{qr} \frac{Q_r \psi_{pr}}{j\omega - \lambda_r} \quad (2)$$

From equation (2), the enhanced frequency response function can be obtained as

$$eFRF_r(\omega) = \frac{Q_r \psi_{pr}}{j\omega - \lambda_r} \quad (3)$$

Equation (3) gives the definition for  $r$ th order eFRF. From this equation, it can be known that natural frequency and modal damping can be identified by

applying eFRF without considering the issue of the magnitude of  $\psi_{pr}$ .

## 2. Strain modal analysis

Starting from strain frequency response function (SFRF), its expression can be written as [12]

$$H_{pq}^{\varepsilon}(\omega) = \sum_{r=1}^{2N} \frac{Q_r \psi_{pr} \varphi_{qr}}{j\omega - \lambda_r} \quad (4)$$

where:  $\varphi_{qr}$  is the  $r$ th strain mode shape for input DOF  $q$ .

Through the definition of strain FRF, all parameters keep the same compared to those of traditional FRF in equation (3), except for strain mode shape under input DOF  $q$  instead of mode shape under input DOF  $q$  in traditional FRF. That means system poles values for strain modal analysis including natural frequency and modal damping can be obtained by performing CMIF method, since the eFRF functions are the same for both traditional FRF and SFRF.

Singular value decomposition technique can be introduced into modal analysis domain, because FRF matrix can be rewritten as [13]:

$$H(\omega) = USV^T \quad (5)$$

where:  $S$  is the singular value matrix;  $U$  and  $V$  are orthogonal matrices with unity length, which satisfy  $UU^T = I$  and  $VV^T = I$ .

Similarly, SFRF can be also expressed as the same way

$$H^{\varepsilon}(\omega) = U^{\varepsilon} S V^T \quad (6)$$

Where:  $U^{\varepsilon}$  comes from singular decomposition of SFRF, the expression manifests the feasibility for identifying strain mode approximately by utilizing SVD technique.

However, another easy and direct approach which can be considered for extracting the strain mode shape is picking up the peak value for SFRF under each mode order.

## II. SIMULATION VALIDATION

A 24 elements and 25 nodes clamped-clamped beam has been simulated by the commercial software ABAQUS. Strain frequency response function for 25 nodes have been extracted by performing harmonic response analysis. The detailed dimension is 1500mm\*40mm\*15mm (length, width and thickness respectively). In order to calculate CMIF, two different loading situation should be performed in the same excitation node. Fig.1 shows the CMIF that there are 5 modes within 500Hz, with sampling frequency 1000Hz. Two different CMIF can be observed due to two different loading condition. Basically, more CMIFs could be observed by applying more loading situations.

The physical parameters are given as following: Elastic modulus  $E$  is  $2.06 \times 10^{11}$  Pa, material density is  $7980 \text{ kg/m}^3$ , and Poisson ratio is 0.3.

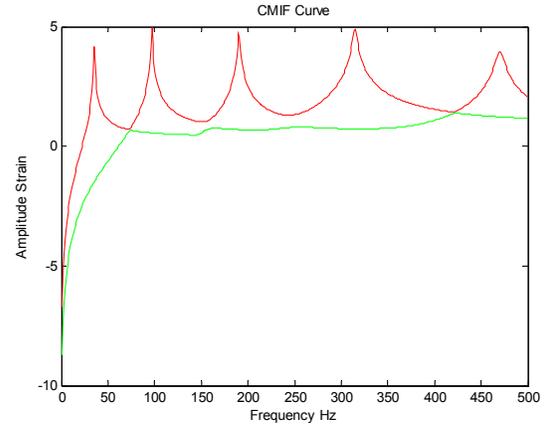


Fig.1. CMIF

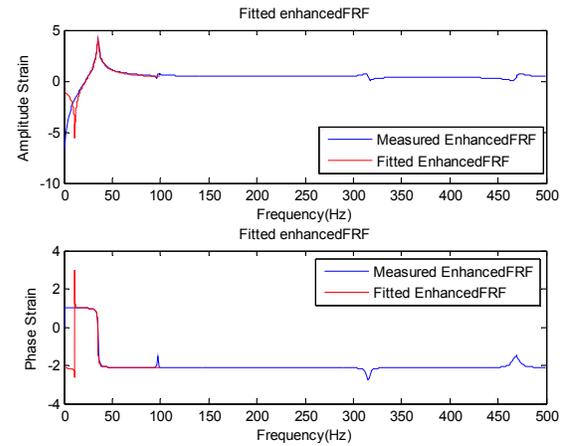


Fig.2. Measured eFRF and fitted eFRF

Only first mode has been chosen, then using the method of single degree freedom to fit the measured eFRF in order to identify modal parameters including natural frequency and modal damping shown in Fig.2.

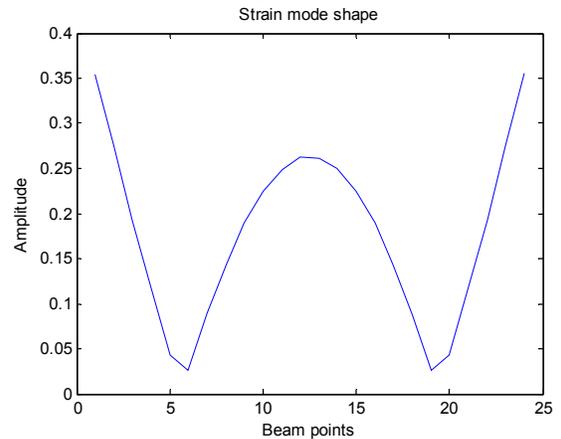


Fig.3. The first strain mode shape

The identification process have been made in Matlab

code, and the identification result for the first mode is given as following: natural frequency 35.21Hz and damping 0.01.

Moreover, the strain mode has been extracted from SFRF by applying SVD technique, and it is shown in Fig.3.

### III. EXPERIMENTAL VALIDATION

#### 1. LUNA-ODiSI-B

LUNA Optical distributed sensor interrogator (Model ODiSI-B) is capable to cover more than 20 meters of dynamic measurement range with a high density of measurement points. Therefore, more accurate and complete dynamic characteristics can be obtained by applying distributed fiber optics which is gradually becoming candidate sensors for purpose of structural health monitoring. ODiSI B can simultaneously demodulate thousands sensing points over a single optical fiber simultaneously at a rate of up to 250 Hz. 20 m maximum sensing distance and spatial resolution of 1.28 mm make ODiSI B an extremely important tool as for strain and temperature sensing applications. Fig.4 shows the operational interface of data acquisition software for ODiSI-B.

#### 2. Experimental setup



Fig.4. Model ODiSI-B



Fig.5. Interface of DAQ system

Here demonstrates the detailed experiments setup. Clamped and clamped steel beam specimen has been installed on the base fixedly, with the dimension 1.8 m × 0.04 m × 0.015 m (where 0.15 m is leaving for each end to make this beam fixed). 2 meters distributed fiber optics has been glued into one-side surface of the beam shown in Fig. 6. The physical parameters are considered as the same as the model in simulation we used.

#### 3. Description of experiment

Two different loading conditions are considered by using hammer in order to generate impulse excitation signals for the beam structure. The configuration of the fiber we use is given here: 2 meters long, with sampling frequency 100Hz and 2.56mm sensing spacing. Herein 497 transducers are taken into consideration.

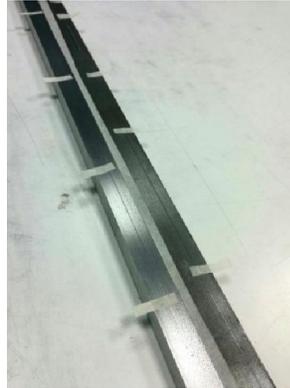


Fig.6. Beam with fiber optic



Fig.7. Beam experiment setup

Fig.7 shows the whole frame of beam structure. The duration of acquisition is lasting for 40 seconds, and exponential window has been applied into the impulse response of the beam in order to reduce leakage.

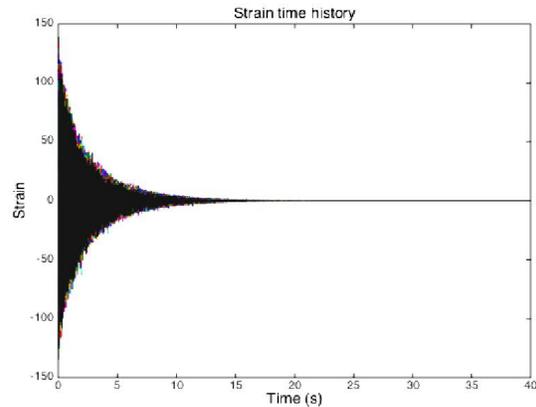


Fig.8. Response in time domain of 497 transducers

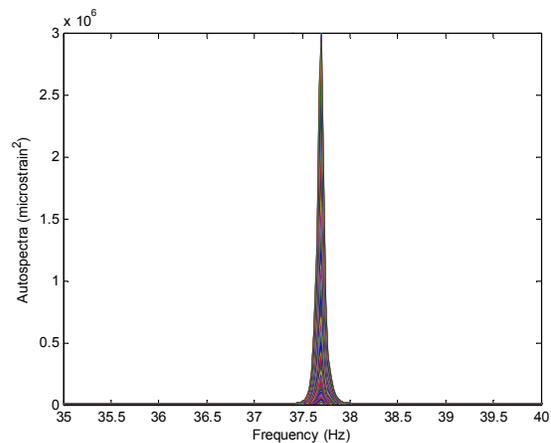


Fig.9. Autospectra of output all the transducers along the fiber

Fig.8 shows the response that acquired by 476 fiber optic transducers in time domain under impulse excitation. The corresponding autospectra are shown in Fig.9. Obviously there is only one mode within 40 Hz.

Modal parameters are able to be recognized through applying CMIF approach which are given in the following: Natural frequency 37.7 Hz, damping 0.016. Considering the uncertain operational and structure setting factors in real experiment, there exists a tolerable deviation in natural frequency and damping compared to the identification result from simulation.

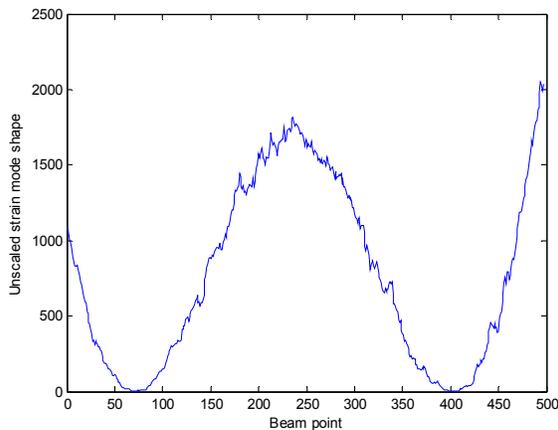


Fig.10. Strain mode shape

Strain mode shape have been obtained by performing SVD technique on SFRF matrix. Due to huge number of sensors (497 sensors in total), strain mode shape observed in Fig.10 is more continuous and clear than that of Fig.3. The asymmetry of both ends is due to the inaccurate experimental setting of fiber configuration along the beam. Generally speaking, CMIF and SVD is able to identify strain modal parameters in a good performance.

#### IV. CONCLUSION

CMIF and SVD can be used into identify the strain modal parameters including natural frequencies, modal dampings and strain mode shapes. Corresponding simulation and experiment shows its effectiveness. Meanwhile, distributed fiber optics have been applied into acquiring response data from a clamped-clamped beam, which contains huge and dense measurement transducers and provide more accurate vibration information.

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