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IDENTIFICATION OF DIAGNOSTIC STATISTICAL STATE MODELS FOR NETWORKED SYSTEM

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Abstract – The diagnostics of networked systems has concentrated on diagnostics of network nodes. We present a statistical modelling method which supports analysis and diagnostics of the network as whole, in particular, of coherent states and state transitions. We describe nodes with discrete states statistically affected by external load. In a network the effective load on a node is a sum of the external load and an effect due to the states of neighbouring nodes. Such effective loads introduce statistical dependence between states, and eventually coherent behaviour of the network. This presentation describes how such models are identified from data and discusses how the models support network-wide diagnostics.

Keywords: networked systems, statistical models, Ising model, diagnostics.

1. INTRODUCTION

Networks for mobile telecommunication, power grids and supply chains are examples of technical networked systems. The importance of networked systems is expected to increase rapidly as the technical and the socio-economical environment is becoming increasingly complex.

Networked systems consist of nodes carrying out operative production or service tasks. Interactions between nodes are due to exchange of material, utility, messages, information, or control actions. The technical diagnostics of networked systems has concentrated on diagnostics of single nodes [1] but the diagnostics of the network as whole and its coherent phenomena is a rather new research topic.

In this paper we outline statistical models with extremely simple description at node level but capable of analysing coherent phenomena and describing joint probability distributions of node states in large networks. With such models it is possible to address questions, such as:

- Is the network as whole in an exceptional state requiring corrective actions (generalisation of multivariate statistical process control to networked systems)?
- How large increase and how spatially distributed load on network would turn the networked system into a qualitatively different coherent state?

The presenting model is based on the Ising model of ferromagnetism of statistical physics [2]. Model is known to exhibit coherent phenomena, which makes it an appropriate choice for modelling and analysis of networked systems as whole.

The rest of the paper is organised as follows. Section 2 introduces the model structure for modelling the networked systems and discusses the joint probability distribution of the state of the network based on the network model presented. Also the properties of the model are discussed shortly. In section 3 the requirements for data for model identification purposes are presented. The identification of network parameters is discussed and an appropriate identification method is introduced. Section 4 gives two examples of the network model identification from a simulated network data. Examples demonstrate how the identification scheme works and manifest the challenges appearing in identification. Section 5 discusses how the model may be applied in analysis of networked systems and presents ideas about the development of the model in the future.

2. MODEL STRUCTURE

In our model the main variables associated to a node in a network are the state of the node (discrete) and the external action/load affecting on the node (real valued scalar). Throughout this paper we consider the node state to be binary (state values being chosen as s = -1, +1) thus dealing with the simplest possible node representation.

If nodes are not interacting, we define the probability of the state S_i of a node as $P(S_i = s_i) = \exp(\beta \cdot s_i(h_i - h_{0,i}))/Z_1$, where h_i is the external action/load affecting on the node i. $h_{0,i}$ is a threshold load above which state $s_i = +1$ is favoured and below which state $s_i = -1$ is favoured. β tells how rapidly the favouring of one state transforms into favouring of the other as the load crosses over $h_{0,i}$. The normalisation factor $Z_1 = \cosh(\beta(h_i - h_{0,i}))$.

The simplest way of introducing interaction between nodes is to define an effective load on the node consisting of the direct external load and an effect of the neighbouring node states. For example, if a neighbouring node in state s_j = -1 reduces the load of the studied node i by J_{ij} and in state s_j = +1 adds a load J_{ij}, the effective load on node i can be written as

$$h_i^{eff} = \sum_{j \in N(i)} J_{ij} s_j + \beta\left(h_i - h_{0,i}\right) \quad (1).$$

Here the subscript i refers to numbering of nodes, $N(i)$ is the set of neighbours of node i, and the other variables were defined above.

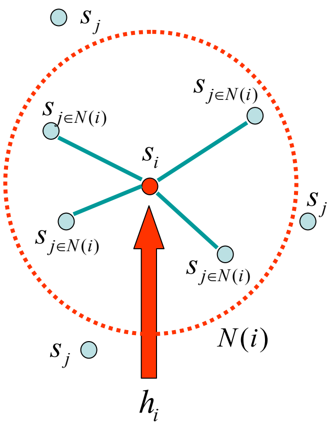

Fig. 1. Example of a network node i, its neighbourhood $N(i)$ and an external load h_i affecting on the node. Nodes of the network are denoted with circles, the neighbourhood of node i with dashed circle and the external load is presented with an arrow.

Fig. 1 exemplifies a network topology and neighbourhood relationships by observing a single network node. The nodes that fall inside the neighbourhood $N(i)$ of node i in Fig. 1 are the neighbouring nodes of the node studied. The external load h_i affecting on node i along with the effect of the neighbouring nodes causes the effective load of (1) on the studied node.

Repeating this analysis consistently to all the N nodes in the network and given the node loads, we generate the joint probability distribution of the states S_i (i = 1,...,N) of the network nodes:

$$P\left[\{S_i = s_i\}_{i=1..N} \middle| \{h_i\}_{i=1..N}\right] = Z^{-1}\exp\left[\sum_{i=1}^{N} s_i h_i^{eff}\right] \\ = Z^{-1}\exp\left[\sum_{i=1}^{N}\sum_{j\in N(i)} J_{ij}s_i s_j + \beta\sum_{i=1}^{N} s_i(h_i - h_{0,i})\right] \qquad (2).$$

This is our statistical network model, closely related to the Ising model of ferromagnetism in statistical physics [2] also used recently in image analysis [3]. The Ising model is known to exhibit coherent phenomena as the size of the 2-dimensional network grows infinite. This is why it is particularly suitable for modelling the phenomena of the network as whole, and chosen here for these purposes.

The topology of the network may either be random or organised in some regular structure, e.g. in regular triangle or square lattice networks, see Fig. 2.

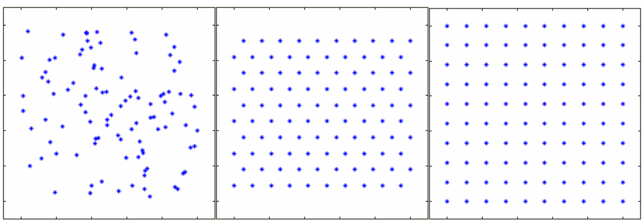

Fig. 2. Topologies of networks: random (left), regular triangle (middle) and regular square (right).

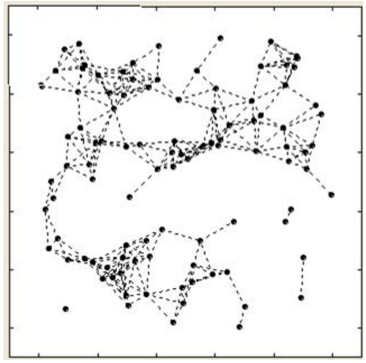

Fig. 3. Example of a network and its neighbourhood relationships. Nodes denoted with dots, neighbours connected with dashed line.

For networks on regular lattice the neighbourhood of each network node defines the topology of the network. With network on a regular lattice we use the concept of nearest neighbours, which means that the neighbourhood consists of the nodes at the distance of a lattice constant to the studied node. However, with random topologies – the more general case – the definition of node neighbours is based on some distance measure, e.g. Euclidean distance. The neighbours of a studied node are all the nodes within a defined distance. Neighbour relations are illustrated in Fig. 3 for a network of random topology.

Throughout the rest of this presentation we assume neighbourhood relationships to be symmetric:

$$j \in N(i) \Rightarrow i \in N(j) \qquad (3).$$

We also assume the threshold load to be independent of the node: $h_{0,i} = h_0$. We shall concentrate on the case where $J_{ij} = J$; a constant throughout the network when nodes i and j are neighbours. In this paper the topology of the studied network is the regular square lattice. However, we shall also discuss more general cases.

3. DATA REQUIREMENTS AND PARAMETER IDENTIFICATION

Network data consists of the state–load pairs defined for all the nodes of the network. Through observing the network over time with certain parameters, we generate a data set

$$\left\{\{s_i^{(l)}\}_{i=1..N}, \{h_i^{(l)}\}_{i=1..N}\right\}_{l=1..L} \qquad (4),$$

where superscript l refers to the numbering of observations and L is the total number of network observations. In practice, the node state data is derived from some more detailed measurements of the node, through e.g. a clustering based classification [1,4]. In this paper, however, the data for identification tests of network parameters is generated with Markov Chain Monte Carlo methods (MCMC) [5] assuming a model structure of (1).

On the basis of this data we want to estimate the network parameters J, β and h_0, thus assuming the network topology defined as known neighbourhood relationships. The normalisation factor Z – needed for estimating the network parameters with maximum likelihood method (ML) [6] – is extremely difficult to calculate for even medium-sized networks [7, Yang-Onsager solution]. Therefore we estimate the parameters based on conditional probability distributions

$$P\left[S_i = s_i \mid \left\{S_j = s_j\right\}_{j \in N(i)}, h_i\right] \qquad (5).$$

Notice here that the conditional probability distribution of node i depends only on the states of its neighbouring nodes. Estimating the parameters through equally weighed sum of logarithms of likelihoods, this leads to the following ML-problem:

$$\begin{aligned} &\arg\min_{J,\beta,h_0} \prod_{l=1}^{L} \prod_{i=1}^{N} P\left[s_i^{(l)} \mid \left\{s_j^{(l)}\right\}_{j \in N(i)}, h_i^{(l)}, J, \beta, h_0\right] \\ &= \arg\min_{J,\beta,h_0} \sum_{l=1}^{L} \sum_{i=1}^{N} \left\{s_i^{(l)} h_i^{eff,(l)} + \ln\left(2\cosh\left(h_i^{eff,(l)}\right)\right)\right\} \end{aligned} \qquad (6)$$

with

$$h_i^{eff,(l)} \equiv J \sum_{j \in N(i)} s_j^{(l)} + \beta\left(h_i^{(l)} - h_0\right) \qquad (7).$$

Here $i = 1,\ldots,N$ refers to a node in the network and $l = 1,\ldots,L$ to the number of network observation.

Next chapter will show examples demonstrating with simulated data that the presented identification scheme – based on conditional probabilities instead of joint probabilities – works assuming the data covers wide enough range of external load situations.

4. EXAMPLES

Identification scheme of the network parameters, based on (6), was tested with two network cases. In the first case each node of the network was influenced by external load which was constant and equal through all the network nodes. The magnitude of the external load was increased deterministically and linearly as a function of network observations from 0 to 1 with even gap of $1/L$, where L is the number of network observations. In the second case, the external loads were changed randomly from node to node and also between network observations. Node loads were generated randomly from a uniform distribution in the interval [0,1] with each network observation. For both identification test cases total of L = 2000 network observations were generated for each parameter configuration. Network size of N = 36 nodes were used with network nodes organised in regular square form, see Fig. 4.

MCMC method was used to generate network data for parameter identification. For simulation the external loads affecting on each node and the network parameters were both fixed. With selected loads and parameters the respective states of the nodes were then simulated with MCMC method. This scheme was repeated several times to generate all the 2000 network observations for each parameter configuration to be used for testing if the identification scheme recovers the network parameters.

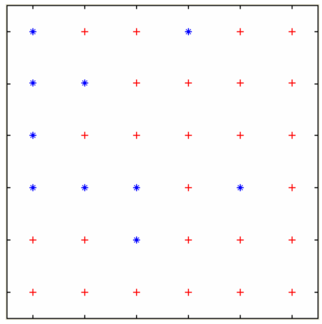

Fig. 4. Example of a network on a regular square lattice. Asterisk (*) denotes state -1 and plus (+) state +1.

The success of identification of parameters J, β and h_0 depends on the ratio of the terms in (7), presented as the ratio of average values of terms over all the network nodes in all the network observations:

$$R = \frac{\frac{1}{K}\sum_{k=1}^{K}\left|\sum_{j \in N(k)} s_j\right|}{\frac{1}{K}\sum_{k=1}^{K}\left|\left(h_k - h_0\right)\right|} = \frac{\sum_{k=1}^{K}\left|\sum_{j \in N(k)} s_j\right|}{\sum_{k=1}^{K}\left|\left(h_k - h_0\right)\right|} \qquad (8).$$

Here k is the number of network node in some network observation: $k = 1,\ldots,N \cdot L$, where N is the total number of network nodes and L is the total number of network observations.

If the ratio of (8) is much larger than 1, then presumably β and h_0 are hard to identify. This is due to the situation when the numerator of R is much larger than the denominator, and the parameters β and h_0 have only a minor effect on the sum in (2) and hence on the state distribution. Respectively, when the ratio of (8) is much less than 1, identification problems with parameter J are expected due to the external load which dominates. In this paper we are interested in the identifiability of the network parameters when coherent phenomena exist; we study the identification of network parameters only with large values of R, i.e. when the interconnection J between network nodes is strong.

Identifiability of the network parameters was tested with a wide range of values of parameters J and β with both previously mentioned test cases. Threshold parameter h_0 was kept constant (0.3) over all the network observations. J and β were given values in the interval [0.1,1] with a step of 0.1. Identification was tested with all the parameter combinations, i.e. total in 100 network parameterisations. The goal was to find out if the identification scheme works and how the success of identification depends on the parameter values of the network.

4.1. First test case: equal load over network nodes

In the first test case all the nodes of the network were affected by equal external loads. Equal loads were increased linearly from one network observation to another, and a sample of network state distributed according to model of (2) was MCMC generated for each loading. The corresponding network parameters were then identified from the generated data and compared with the known original ones.

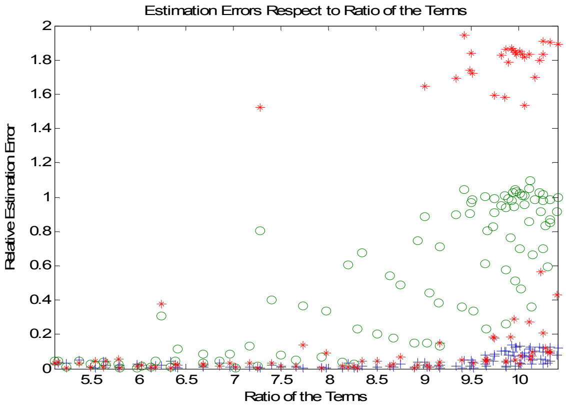

Fig. 5. Relative parameter estimation errors with each parameter configuration as a function of R of (8). Plus (+) denotes the errors of parameter J, circle (o) of β and asterisk (*) of h_0.

In general, the identification of the network parameters succeeded quite well. However, when the ratio R of (8) becomes large, some identification problems occur. This is illustrated in Fig. 5, where the relative estimation errors of each network parameter are presented as a function of R. That is, we plot the set of data pairs

$$S_1 = \left\{ \left| 1 - \frac{\hat{y}_p}{y_p} \right|, R_p \right\}_{p=1}^{P} \tag{9}$$

for each network parameter J, β and h_0 separately. Here $\hat{y}_p$ denotes the parameter estimate and y_p the true value of the parameter. R_p is the ratio value defined in (8) and p is the number of parameter combination. Here we have total of $P = 10^2 = 100$ parameter combinations.

Fig. 5 shows that when the R of (8) increases the effect of β and h_0 on the optimisation function (6) decreases and the estimation of these parameters becomes more difficult. Also, the success of estimation of J does not seem to depend much on R. As the figure shows, the values of the ratio R are quite large, i.e. the effect of the neighbouring nodes is always much greater than the effect of the external load on the node studied. This favours the identification of J in the first term of (2), and the identification of J is successful.

Fig. 6 presents the same relative estimation errors as Fig. 5 for each parameter combination (J, β, h_0). Now the values in x-axis are the variances of the network states between all the network observations. By the state of the network, we imply here to the mean value of the node states over all the network nodes in certain network observation. That is, Fig. 6 plots the set of data pairs

$$S_2 = \left\{ \left| 1 - \frac{\hat{y}_p}{y_p} \right|, \operatorname{var}\left\{ \frac{1}{N} \sum_{i=1}^{N} s_{ilp} \right\}_{l=1}^{L} \right\}_{p=1}^{P} \tag{10},$$

where we have used the notation of (9). Here i index the number of network node in certain network observation l.

Fig. 6 is similar to Fig. 5; the larger the variances of the network states between the observations are the more difficult it is to estimate the parameters. This is due to the week representativeness of the data with a given parameter configuration. If the network data varies a lot between observations with certain parameters, then the effect of the parameters on the network state is uncertain and the identification is hard. We want to note that a single data point fall outside of Figs. 5–6, with a relative error of ~ $8.4 \cdot 10^5$ and R ~ 9.7.

According to (8) it is rather clear that the success of estimation of each parameter should depend also on the magnitudes of the values of the other parameters. For example, if the value of J is large then the estimation of β and h_0 should become harder. Respectively, if β is large then the identification of J should be more difficult. This is illustrated in Fig. 7, where the relative estimation errors of each parameter are presented in the three down most graphs. The two top most graphs depict the true values of J and β with each parameter combination. Recall that the value of h_0 is constant through all the parameter identifications. Figure describes how the success of the identification depends on the true values of J and β. The x-axis contains the 100 parameter combinations used for identification tests.

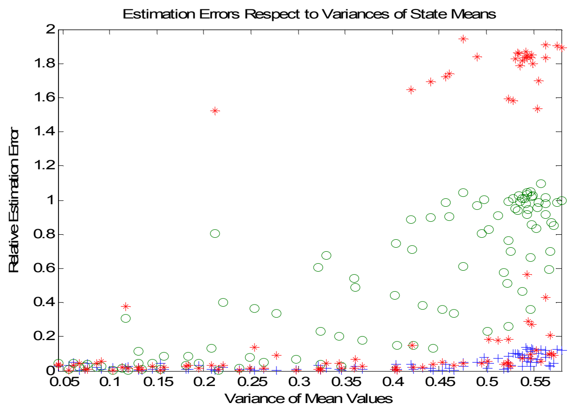

Fig. 6. Relative parameter estimation errors with each parameter configuration as a function of variance of network state between network observations – the set of data pairs defined in (10). Plus (+) denotes the errors of J, circle (o) of β and asterisk (*) of h_0.

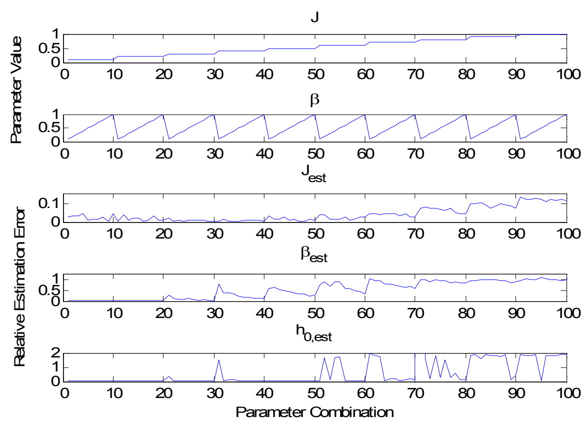

Fig. 7. The true values of parameters J and β in the two top most graphs. Relative parameter estimation errors of J, β and h_0 in the three down most graphs as a function of parameter combinations.

Fig. 7 shows that the identification of J is quite successful through all the parameter combinations, and the relative estimation error is rather small. However, some difficulties arise when the true value of J becomes large; the relative estimation error seems to increase according to Fig. 7. The true values of parameter β do not seem to have much effect on the identification of J. However, the relative estimation error of β seems to become larger when the value of J increases. Also, the increasing values of the true β may have slightly positive effect on the identification of the parameter itself and also on the parameter h_0. This seems logical because large values of β compensate possible large values of R of (8). Fig. 7 also shows that the identification of h_0 is difficult with large values of J.

4.2. Second test case: random loads over network nodes

In the second test case the external loads affecting on the nodes of the network were selected randomly for each network node. Random selection was based on uniform distribution in interval [0,1]. Random selection of node loads was repeated with each network observation. The generated data was again used for identification of the network parameters.

Fig. 8 presents the set of data pairs S_1 defined in (9) for the second test case. Again, the relative estimation errors increase as a function of R of (8). However, compared to the first test case, there are two particular differences in the identifiability of the parameters: firstly the identification of β is now clearly more successful and secondly the problems in identifying h_0 increase nearly linearly as a function of R.

Fig. 9 presents the resulting relative estimation errors, the set of data pairs S_2 defined in (10). The difference between Figs. 8 and 9 is more obvious than the difference was between Figs. 5 and 6. However, the form of the two figures is similar; estimation errors become larger when the ratio of (8) and the variance of network states increase. Altogether, there seems to be clear dependence between the variance defined in (10) and the ratio R presented in (8). We want to note that three data points fall outside of Figs. 8–9 and had relative errors of around 2 with ratio values of 9.3–9.9.

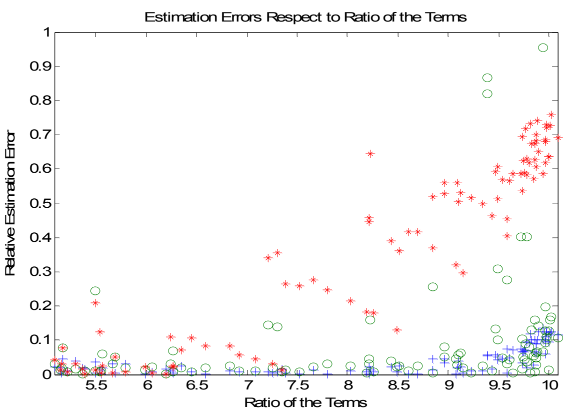

Fig. 8. Relative parameter estimation errors with each parameter configuration as a function of R of (8). Plus (+) denotes the errors of parameter J, circle (o) of β and asterisk (*) of h_0.

In Fig. 10 the relative estimation errors of J, β and h_0 are presented in the three down most graphs, while the two top most graphs depict the true values of J and β with each combination of the parameters. According to Fig. 10 identification of β does not seem to depend on the true value of J as clear as with the first test case, although some increase in the estimation error can be observed while J is increased. This error also seems to be fractionally larger when the value of β itself is small. This was already discussed shortly when analysing Fig. 7. Observations from the identification of h_0 are very similar with β above. However, the identification of h_0 depends more heavily on the values of J.

Fig. 10 shows that, as before in Fig. 7, the identifiability of J depends on its true value – the relative estimation error increases as J becomes larger. One could assume better identifiability of J with large values. However, when J becomes large network nodes prefer coherent, network-wide states. In our network simulations each node was initially in state -1, and with large J (> 0.5) the network prefers to stay in this state nearly regardless of the external loads. The impact of J is very similar through its large values and with all the observations. In lack of clear dependence between J and the network state the identifiability degrades.

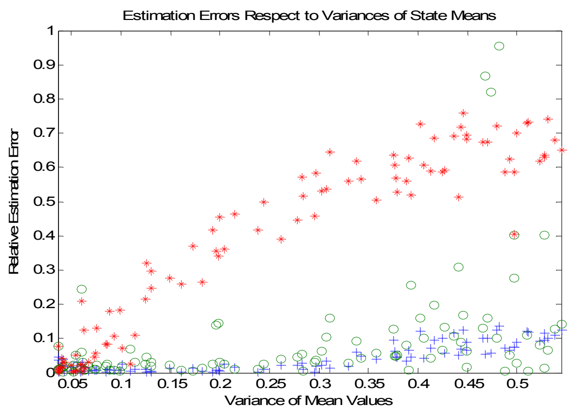

Fig. 9. Relative parameter estimation errors with each parameter configuration as a function of variance of network state between network observations – the set of data pairs defined in (10). Plus (+) denotes the errors of J, circle (o) of β and asterisk (*) of h_0.

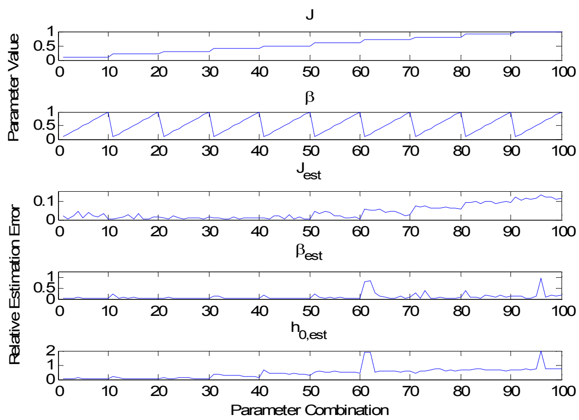

Fig. 10. The true values of parameters J and β in the two top most graphs. Relative parameter estimation errors of J, β and h_0 in the three down most graphs as a function of parameter combinations.

5. CONCLUSIONS

In this paper we presented an identification scheme for statistical modelling of a networked system with a model closely related to the Ising model. The presented identification scheme was based on the conditional distributions of the node states of the network defined in (6). The two identification test cases showed how the presented identification scheme works. We also discussed the problems that may occur in identification and introduced a term ratio R of (8) which indicates possible difficulties in identification when R gets large values.

The presented statistical model is based on a very simple node state description but it is very suitable for describing the phenomena of the network as whole: the model can be effectively used for analysing coherent phenomena and describing joint probability distributions of node states in large networks. With such models we may study if the modelled network is in an exceptional state. We may also study weather the network undergoes a coherent phenomena turning it into a qualitatively different state when spatially distributed load is increased at certain point of the network.

With an identified statistical model presented here the network response to external load can also be simulated. Network states are generated with Markov Chain Monte Carlo method. In this method at each step a node is selected randomly and rest of the network is frozen. A new random state is obtained according to the conditional distribution of (5). After sufficiently many steps the network state is distributed according to the joint probability distribution of the network states defined in (2). With such simulation scheme we may simulate and study the phenomena of real networks with the identified statistical models describing them. Fig. 11 shows an example of network state simulated with certain parameter and load values. Figure presents the node states corresponding to neighbourhood relations defined in network of Fig. 3.

The presented modelling method is being developed for monitoring and diagnostics of mobile telecommunication networks. At present, we have derived a parameter identification scheme under rather strong simplifying conditions and for simulated data. Our next steps will be to apply the identification scheme to network which nodes are randomly organised. We shall also extend the number of discrete states of network nodes and apply the method to real GSM network data in which the node state data is obtained through pattern recognition of Key Performance Indicators (KPIs). KPIs are based on numerous counters continuously accumulated within base stations.

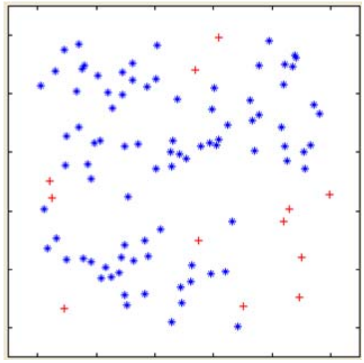

Fig. 11. Example of simulated state of the network of Fig. 2. Asterisk (*) denotes state -1 and plus (+) state +1.

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