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Quantization Noise of the Non-uniform Exponential Tracking A/D Conversion

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Abstract - The paper presents the possibility of an adaptive A/D conversion with the non-uniform exponential tracking procedure. It has been shown that the proposed A/D conversion gives better results than the classical A/D conversion with the successive approximation procedure due to *b*-times more available sampling points. Taking into consideration only the quantization noise contribution the adaptive A/D conversion performs better results if the sampling ratio s is high enough.

I. Introduction

Estimation of the measured quantity in technical system has the discrete nature in amplitude and time dimensions. In the majority of different structures of A/D conversions [1]-[3], a numerical value of the measured quantity is attained with reducing of the error E_G between the auxiliary reference quantity G_R and the real value of the measured quantity G (Fig. 1.: $E_G = G_R - G$). The residual error is reduced through the estimation procedure in steps and final it attains the basic resolution of conversion Δ_0. Here a trade between the number of references for generating the reference levels and the number of steps of the conversion is presented. Optimal results for the A/D conversion with regard to the time of conversion, resolution, and used references are obtained with the multi-step parallel techniques [4]. Since they estimate the residual error with more threshold levels then the pure successive approximation procedure with one threshold level in every step (Fig. 3b.) they are faster and at the same time they need a lower number of references for threshold levels in comparison to the pure one-step parallel flash technique due to more steps in the estimation procedure. The threshold levels can be arranged in the non-uniform exponential rule to increase the quantization resolution at the origin of the examination of the residual error [2] [7] (Fig. 2.).

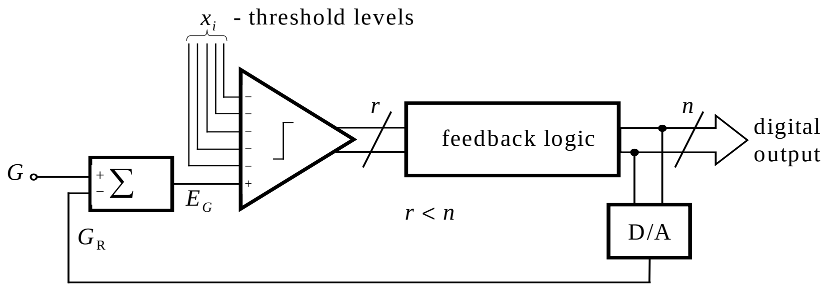


Figure 1. Differential tracking converter

II. Non-uniform quantization

For effectiveness of differential tracking, the non-uniform quantization must fulfil three conditions: partitions into halves [5], increasing quantization uncertainty with difference [6], and low overlapping of the quantization intervals. The best trade between the number of decision levels and the settling time is with the pure exponential quantization rule. The fastest response is achievable with base 2 [7]. The exponential distribution function of representatives y_i with base $a = 2$ has several advantages: simplicity of the mathematical implementation, exponential emphasising on the surroundings of origin, and possible practical realization [2].

$$y_i = a^{i-1}\Delta_0 = 2^{i-1}\Delta_0 \qquad i = 1, 2, \ldots, b \tag{1}$$

There must be a symmetry around the origin y_0 (Fig. 2) to attain effectiveness of quantization (minimum distortion) and the fastest response of conversion. Index i of representatives obtain a negative sign, but their distance to y_0 remains the same $|y_i - y_0| = |y_{-i} - y_0|$.

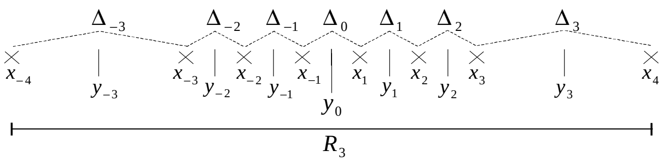


Figure 2. Representatives y_i and threshold levels x_i of the non-uniform exponential quantization for $i = 3$ and base $a = 2$

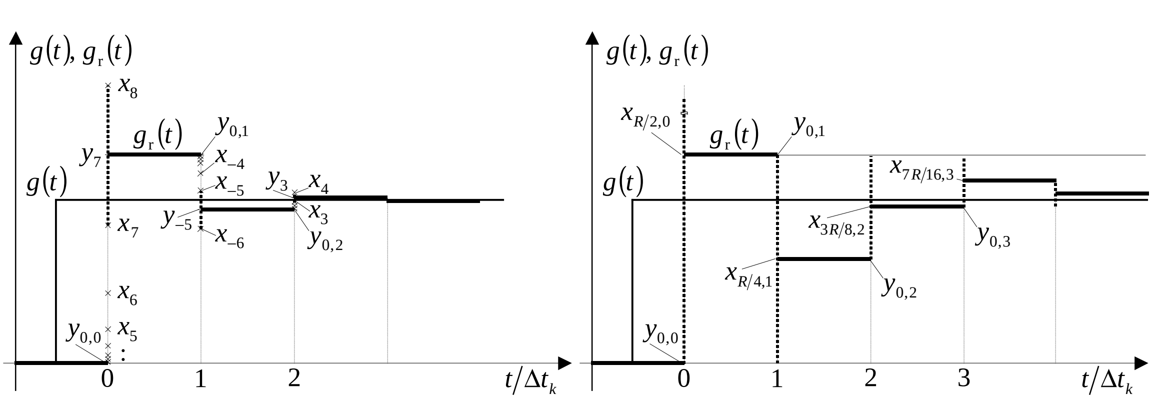
The first threshold - decision level x_1 must lie in the middle between y_1 and y_0 ($x_1 = \Delta_0/2$). This assures that the smallest quantization interval Δ_0 is that around the origin ($\Delta_0 = R/2^b$; R - full scale of the measurement range, b - a number of bits). Other decision levels can be obtained by induction, if the generic function $x_{i+1} = 2y_i - x_i$, x_1, and y_1 are known. The expression for x_i ($a = 2$) can be written as:

$$\frac{x_i}{\Delta_0} = (-1)^i \left(2\,\frac{1-(-a)^{i-1}}{1-(-a)} - \frac{1}{2} \right) \underset{a=2}{=} (-1)^i \left(\frac{2+(-2)^i}{3} - \frac{1}{2} \right) \tag{2}$$

The distance of y_i from y_0 increases the quantization error of the representative $\Delta_i/2$ or the quantization interval Δ_i. The quantization interval is fixed with the difference of adjacent thresholds x_i.

$$\frac{\Delta_i}{\Delta_0} = \frac{(x_{i+1} - x_i)}{\Delta_0} = \frac{2a^{i-1}(a-1)}{1+a} + (-1)^{i+1}\frac{3-a}{1+a} \underset{a=2}{=} \frac{1}{3}\left(2^i + (-1)^{i+1}\right) \tag{3}$$

With the adaptive property of the A/D procedure that every previous approximation step to signal become the centre of observation with exponential increasing resolution in the new step is possible to short the time of conversion. The adaptive A/D converter is possible to use in two different ways: with and without sample/hold device. In the first case the converter approaches with a non-linear proceeding of reducing uncertainty to the constant measured value (Fig. 3.a). Since steps y_i are exponentially interspaced with base $a = 2$ it is easy to logically implemented one step by increment or decrement the value in the register of the feedback logic by 1 at the suitable weighted bin ($000_{\updownarrow y_i} 1_{\updownarrow y_{i-1}} 110...$). Each conversion is finished with uncertainty of the smallest quantizing interval $\pm\Delta_0/2$. In comparison with the successive-approximation method (Fig. 3b.) the time of conversion is shortened in average to one third or at least to half by two reasons [7]. The difference between approximation and real value is measured in every step and it is not necessary to begin testing the most significant bits, if the new value at input of converter does not greatly differ from the previous one.



a. b.

Fig. 3. Step response of the approximation procedure of the error reduction between signal g and reference quantity g_r : a –the non-uniform approximation procedure; b - the successive approximation procedure

Proposed A/D converter works also without sample/hold device (Fig. 4). Part of the sample/hold function takes over the latch register in the logic of the feedback path. It holds the value of previous approximation $g_r(t_{k-1})$, until the estimation $\langle E_G \rangle = y_i$ of difference between the new value of measured signal and the previous approximation is available after $\Delta t_k = t_k - t_{k-1}$.

$$g_r(t_k) = g_r(t_{k-1}) + y_i(t_k) \tag{4}$$

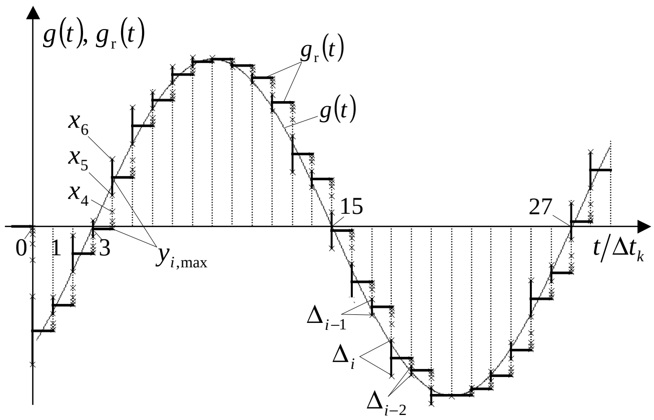


Fig. 4. Estimation and tracking of the sine shape signal with 24 samples in the period

Tracking of the signal is achieved by the non-uniform estimation of the error difference $\langle E_G \rangle$ and increasing the value of the reference quantity by suitable step y_i. Estimation of the signal at time instant t_k has uncertainty of the quantization interval Δ_i which belongs to y_i. The largest are at the zero level crossings of the AC signals.

III. Quantization noise

The quality of tracking of the sine shape signal depends mainly on the number of samples in one period $s = T/\Delta t_k$. The lower is the sampling ratio *s* the larger is the signal change in one time increment Δt_k and consequently the non-uniform increment y_i (Fig. 4). Increasing of the sampling ratio *s* decreases the largest step $y_{i_{\max}}$ (Fig. 5). The reduction of the largest step with index $i_{\max}$ was tested by the sine-shape signal (with amplitude $A = R/2$ for the worst case). Since the number of the A/D converter bits was $b = 12$ the first six settling steps of the tracking was removed and after that four cycles of the signal were examined. To find the maximal possible step $y_{i_{\max}}$ at every sampling ratio *s* the phase of the sine signal was changed $0 \le \varphi \le 90^\circ$ with resolution $\Delta\varphi = 1^\circ$. The first reduction of the largest step $y_{i_{\max}} = R/2$ to $y_{i_{\max}-1} = R/4$ is at $s = 11_{i_{\max}-1}$ samples in the period. The next reductions are at $s = 24_{i_{\max}-2}, 48_{i_{\max}-3}, 98_{i_{\max}-4}, 198_{i_{\max}-5}, 398_{i_{\max}-6}, 797_{i_{\max}-7}, 1602_{i_{\max}-8}, \ldots$. The reduction of the largest steps is independent of the number of bits b.

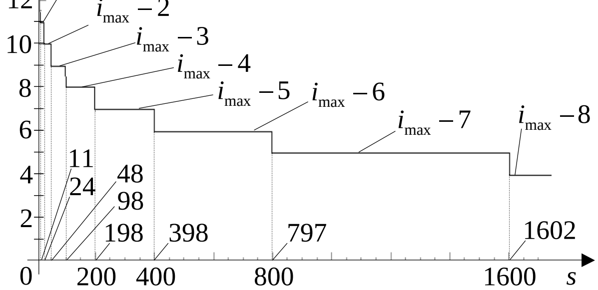


Fig. 5. Reduction of the index of the maximal steps in relation to the sampling ratio *s* of the adaptive ADC

The probability distributions of the steps show (Fig. 6) that practically the largest six steps mainly contribute in tracking.

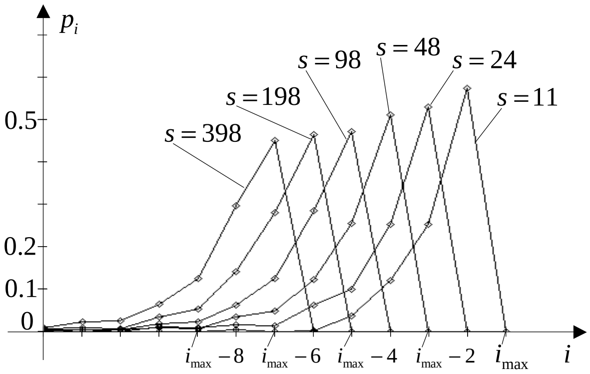


Fig. 6. Probability distributions of the approximation steps in relation to the characteristic sampling ratios *s* of the adaptive AD converter

The largest six steps in tracking and belonging uncertainties $\Delta_i/\sqrt{12}$ increases also quantization noise [8] in comparison to the lowest value $\sigma_0=\Delta_0/\sqrt{12}$ (Fig. 7II.). With increasing of the sampling ratio s in tracking is possible to decrease very fast the quantization uncertainty σ_q (Fig. 7, testing conditions are the same as for Fig. 5). It can be noticed that the quantization standard deviation of tracking the signal a little differs from the expected standard deviation using the probability distributions of the approximation steps at the lower values of s (Figs. 7a., 7b.) due to the small number of samples in the particular qunatization intervals and accordingly the expected rectangular error distributions are not achieved.

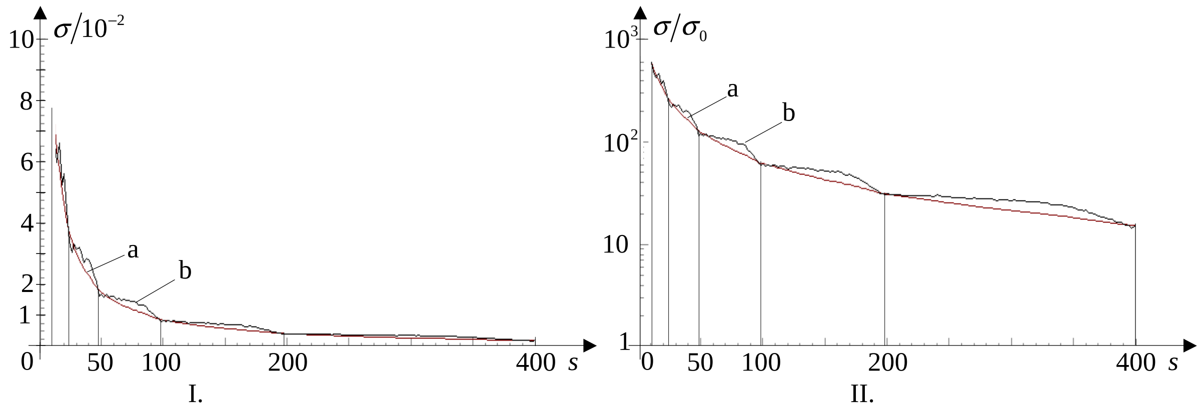


Fig. 7. Standard deviations (I.) and ratios of the standard deviations to $\sigma_0=\Delta_0/\sqrt{12}$ (II.) in relation to the sampling ratio *s* of the 12-bit adaptive AD converter: a – approximation using the probability distributions of the approximation steps like in Fig. 6; b – results of tracking the signal

The increased quantization noise at lower sampling ratios s is at the price of increasing *b*-times the available sampling points in comparison to the A/D converter with the successive approximation procedure which have to perform all *b* steps to the final uncertainty $\Delta_0/\sqrt{12}$ of the smallest quantization interval Δ_0. Since we have *b*-times more sampling points using adaptive A/D converter the noise contribution in the estimation of the sine signal parameter (like amplitude, frequency, and phase) is reduced by $\sqrt{b}$ [8] (Fig. 8). Taking into consideration only the quantization noise contribution in parameter estimations the proposed adaptive A/D conversion performs better results than the successive approximation A/D conversion if the sampling ratio s is high enough (Fig. 8.: $s>634$ for 10-bit A/D converter and $s>1730$ for 12-bit A/D converter).

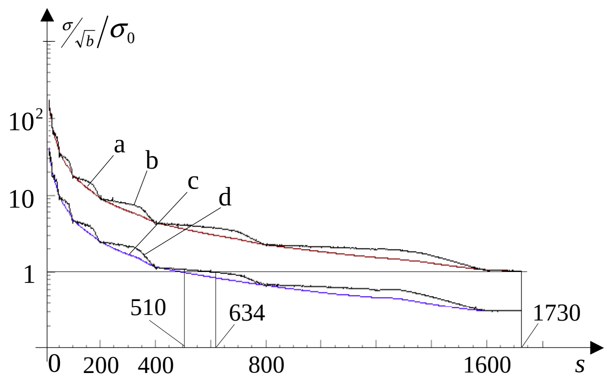


Fig. 8. Ratios of standard deviations of the adaptive AD conversion to $\sigma_0 = \Delta_0/\sqrt{12}$ in relation to the sampling ratio *s*: a – approximation using the probability distributions of the approximation steps with 12-bit AD converter; b – results of tracking the signal with 12-bit AD converter; c – approximation using the probability distributions of the approximation steps with 10-bit AD converter; d – results of tracking the signal with 10-bit AD converter

Considering together both contributions - the systematic and the random errors – in the signal parameters estimations shows the advantage of the adaptive A/D conversion also at lower values of the sampling ratio s. For demonstration, we checked the error of the amplitude estimation $e(A) = A/A^* - 1$ (A^* is the true value of the amplitude) for one sine component with a double scan varying both the relative frequency $\theta = f \cdot T_{\mathrm{M}} = i + \delta$ (the number of cycles in the measurement interval T_{M}) and the phase of the signal because the long-range leakages are frequency and phase depended (Fig. 9.: $A = 1$, $1.6 \le \theta \le 10$, $\Delta\theta = 0.01$ and $-90^\circ \le \varphi \le 90^\circ$, $\Delta\varphi = 3^\circ$, $b = 12$). In estimations, the three-point interpolated DFT algorithms and the Hann window were used [9].

$$\delta_m = 2\frac{|G(i_m+1)| - |G(i_m-1)|}{|G(i_m-1)| + 2|G(i_m)| + |G(i_m+1)|} \tag{5}$$

$$A = \frac{\pi\delta_m}{\sin(\pi\delta_m)}\frac{(1-\delta_m^2)(4-\delta_m^2)}{3}\cdot\left[|G(i_m-1)| + 2|G(i_m)| + |G(i_m+1)|\right] \tag{6}$$

The maximum values of errors (from 60 iterations) at the given relative frequency show the benefit of the adaptive A/D conversion even at lower values of the sampling ratio if the number of the measured signal cycles is small.

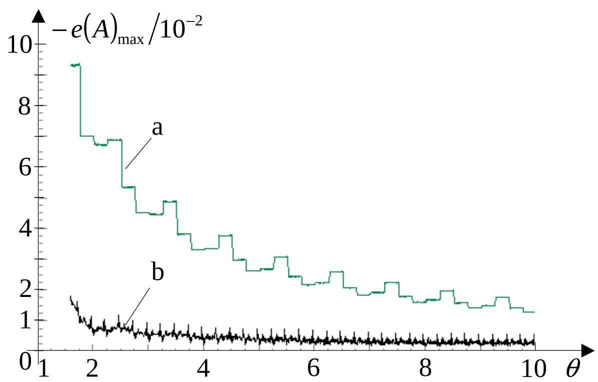


Fig. 9. Maximal errors of the amplitude estimations with 12-bit AD converters in relation to the relative frequency: a – the successive approximation AD converter with $s_0 = 4$ samples per period; b – the adaptive AD converter with $s = b \cdot s_0 = 48$ samples per period;

IV. Conclusions

In this paper, the possibility of an adaptive A/D conversion with the differential tracking procedure of the signal by the non-uniform exponential quantization is presented. In analyses, it has been shown that the proposed *b*-bit A/D conversion gives better results than the classical A/D conversion with successive approximation procedure due to *b*-times more available sampling points. Taking into consideration only the quantization noise contribution the adaptive A/D conversion performs better results if the sampling ratio s is high enough.
Considering together the systematic and the random errors in the signal parameters estimations shows the advantage of the adaptive A/D conversion also at lower values of the sampling ratio s .

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