

# The DTCM characterization approach for the qualification of dynamic non-linearities within A/D channels

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**Abstract**- The behaviour of both stand-alone ADCs and entire A/D acquisition channels under time-varying input excitations exhibits non-idealities which are peculiar to the dynamic operation and can not be described by the characteristics of the static response. In addition, even the common dynamic parameters suggested by the current standards usually fail to separately qualify the purely-dynamic non-linearities from all other perturbation effects on the system response. In this paper, the features of the Discrete-Time Convolution Model are discussed by pointing out the properties of the general approach from which it derives, with particular emphasis to the capability of separately describing all the system non-idealities on the basis of their nature. It will be shown how the DTCM characterization methodology and the set of related model parameters represent a suitable resource towards the separate qualification of dynamic non-linearities in A/D channels.

## I. Introduction

A vast literature has been produced in the last two decades concerning the testing and characterization of analogue-to-digital converters (ADCs) under dynamic operation. Such a research effort has also led to widely known and extensively used standardization documents (e.g., [1,2]), in which a set of figures of merit have been proposed as well, for the qualification of the dynamic distortion at the output of the system and the comparison among different ADCs in terms of dynamic behaviour, at least under reference input excitations (sinusoidal, in most of the cases). However, while it is widely recognized that the dynamic operation of an ADC (or, from a more general standpoint, the overall input channel of a digital data acquisition system) exhibits specific non-idealities at the output, which are not associated with the static response and can not be characterized by quasi-static testing procedures involving DC or slow input signals, both the dynamic techniques and qualifying parameters proposed usually lack the capability of separately describing the contribution of *purely-dynamic* non-linearity on the perturbation on the output of the system.

In the last few years, some papers (e.g., [3-5]) demonstrate the growing interest towards the characterization, modelling and somehow compensation of ADC non-linearities which are peculiar to dynamic operation. A valuable result of such an advanced investigation would also be parameters, which could quantify in a compact but rigorous way the purely-dynamic non-linear phenomena within the system, to be used for supplementing the standard dynamic figures of merit.

In the past, the authors proposed a behavioural methodology (i.e., the Discrete-Time Convolution Model, DTCM [6,7]) for the characterization and modelling of digital data acquisition channels, which is based on a modification of the classical Volterra series. The possibility of compensating the system non-idealities by means of techniques capable of analytically inverting the DTCM non-linear dynamic laws has been also demonstrated [8]. In this paper, a discussion on the analytical properties of the modified Volterra series is provided, as well as on the assumptions which lead to the DTCM operative formulation, in order to show how this characterization methodology and related model parameters are suitable for the definition of metrics and figures of merit aimed at the separate qualification of dynamic non-linearities in stand-alone ADCs or entire A/D acquisition channels.

## II. The modified Volterra series and “short-memory” non-linear systems

The classical Volterra series is one of the best-known methods for the behavioural description of the input/output functional relationship of a time-invariant, non-linear dynamic system [9,10]. Its time-domain formulation, for a system of input  $s(t)$  and output  $y(t)$ , can be written as

$$y(t) = y_0 + \int_{-\infty}^{\infty} h_1(\tau_1)s(t-\tau_1)d\tau_1 + \sum_{r=2}^{\infty} \frac{1}{r!} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h_r(\tau_1, \dots, \tau_r)s(t-\tau_1) \cdots s(t-\tau_r)d\tau_1 \cdots d\tau_r \quad (1)$$

the  $r$ -dimensional function  $h_r$  being the  $r$ th-order Volterra kernel. In the expansion of Eq. (1), the first-order integral contribution to the series has been made explicit, in order to point out how this analytical description can be considered as a generalization of that representing the response of a *linear* system with memory, by means of the single-fold convolution between the input signal and the pulse response. The latter is now interpreted, according to a generalized standpoint, as the first-order Volterra kernel: all the higher-order

kernels contribute instead to describe the overall *non-linear* contribution to  $y(t)$ .

Although the Volterra approach has been widely exploited in the past in many fields of application, its use presents the inherent constraint of requiring complex identification procedures since, even in the presence of weakly non-linear systems, the kernels up to at least the third or fourth order must be experimentally estimated in order to achieve an adequate accuracy in the prediction of the system behaviour. For systems with stronger non-linearity, or applications which require very high levels of accuracy, such as those involving systems operating in measurement set-ups, the Volterra series should be truncated to even higher orders. This can make the classical approach unreliable or even non-feasible. However, independently of the fact that a suitable number of kernels could be accurately identified for a given system or not, Eq. (1) shows that such a series expansion is very suitable for pointing out the separation between linear and non-linear effects within the system, but fails to separately characterize the purely-non-linear *static* (i.e., memoryless, or algebraic) and *dynamic* phenomena. In fact, it could be easily shown that each dynamic, multi-fold integral in the summation in Eq. (1) also provides a contribution to the static non-linear response of the system, which can not be directly appreciated by simply observing the corresponding kernel. To this aim, i.e., in order to separate static and dynamic non-linear responses, a numerical manipulation of the estimated kernels should be needed, with strong issues of inaccuracy mainly due to the aforementioned criticality of the estimation of high-order kernels, which is, moreover, discrete on and limited to sub-planes within their domain.

An equivalent formulation of (1) can be derived by defining the *dynamic deviation* function  $e(t, \tau) \equiv s(t - \tau) - s(t)$ ,  $\tau \in (-\infty, \infty)$ , which is the deviation, with respect to the instantaneous value of the input, of the evolution of the latter along the virtually non-finite “memory-time” of the system [11]:

$$y(t) = z_0[s(t)] + \sum_{r=1}^{\infty} \frac{1}{r!} \int_{-\infty}^{\infty} \int g_r[s(t), \tau_1, \dots, \tau_r] e(t, \tau_1) \cdots e(t, \tau_r) d\tau_1 \cdots d\tau_r \quad (2)$$

with

$$g_r[s(t), \tau_1, \dots, \tau_r] = h_r(\tau_1, \dots, \tau_r) + \sum_{m=1}^{\infty} \frac{1}{m!} s^m(t) \int_{-\infty}^{\infty} \int h_{r+m}(\tau_1, \dots, \tau_{r+m}) d\tau_{r+1} \cdots d\tau_{r+m} \quad (3)$$

Equation (2) represents the *dynamic deviation-based modified* Volterra series,  $g_r$  being the  $r$ th-order modified kernel, which is non-linearly controlled by the instantaneous value of the input  $s(t)$ . Since the summation in (2) is now a *purely-dynamic* contribution (taking into account both non-linear and linear dynamic effects) due to the presence of the dynamic deviations  $e(t, \tau_i)$  in place of the evolutions  $s(t - \tau_i)$  of the entire input signal, the algebraic function  $z_0$  (sometimes referred to as the zero-order modified kernel) coincides with the *static* response of the system. Thus, in such a reformulation, the static and the purely-dynamic responses of the system are separated and inherently made explicit.

The modified Volterra series preserves the generality of the classical approach (1), but also presents two fundamental peculiarities: *i)* very different convergence properties when suitable hypotheses are satisfied about the relationship between the memory-time of the system and the dynamics of the typical signals applied, and *ii)* it not only explicitly points out linear effects from non-linear ones, as in the case of the Volterra series, but the static and purely-dynamic non-linearities are separately described as well. In the following, these two issues will be addressed.

By deriving from Eq. (3) the analytical expression of the first-order modified kernel  $g_1$ , Eq. (2) can be further expanded in

$$y(t) = z_0[s(t)] + \int_{-\infty}^{\infty} h_1(\tau_1) e(t, \tau_1) d\tau_1 + \int_{-\infty}^{\infty} \left( \sum_{m=1}^{\infty} \frac{1}{m!} s^m(t) b_m^{(1)}(\tau_1) \right) e(t, \tau_1) d\tau_1 + \sum_{r=2}^{\infty} \frac{1}{r!} \int_{-\infty}^{\infty} \int g_r[s(t), \tau_1, \dots, \tau_r] e(t, \tau_1) \cdots e(t, \tau_r) d\tau_1 \cdots d\tau_r \quad (4)$$

$b_m^{(1)}(\tau_1)$  ( $m = 1, 2, \dots$ ) being the multi-fold integral over the sub-domain  $(\tau_2, \dots, \tau_{1+m})$  of conventional kernel  $h_{1+m}$ . In Eq. (4), the purely-dynamic linear response of the system (i.e., the second element to the right side) is pointed out. The third and fourth elements to the right side of Eq. (4), which consist of the integral terms of the modified series for  $r \geq 2$  and a single-fold integral involving the input-controlled residual contribution to  $g_1$ , are thus not only purely-dynamic, but also *purely-non-linear*. This completes the aforementioned separation, in the description of the system response offered by the modified series, of different effects on the basis of their nature: from left to right in Eq. (4), purely-static, dynamic purely-linear, dynamic purely-non-linear.

The superposition of the last two terms in Eq. (4) is a non-parametric behavioural description for the dynamic non-linearities associated with the system, which is as general as the Volterra series, but at the same

time is also de-embedded from any other effects of different nature. In order to achieve a modelling approach exploitable in practical applications, it can be observed that in most of the systems of interest the memory-time associated with non-linear dynamics can be truncated to a *finite* interval  $[t - T_B, t + T_A]$  around the instant  $t$  at which the output  $y(t)$  is evaluated. The purely-dynamic non-linear response of the system can be thus adequately described as follows, without any practical loss of accuracy:

$$y^{(D)}(t) = \int_{-T_A}^{T_B} \left( \sum_{m=1}^{\infty} \frac{1}{m!} s^m(t) b_m^{(1)}(\tau) \right) e(t, \tau) d\tau + \sum_{r=2}^{\infty} \frac{1}{r!} \int_{-T_A}^{T_B} g_r[s(t), \tau_1, \dots, \tau_r] e(t, \tau_1) \cdots e(t, \tau_r) d\tau_1 \cdots d\tau_r \quad (5)$$

Moreover, in a still wide range of application, the “non-linear” memory-time duration can be assumed as being “short” when compared to the typical period  $T_0$  of the applied input signals. This is the case, for example, of stand-alone non-linear electron devices [11,12], whose technology is constantly oriented to the fabrication of semiconductor physical structures with ultra-fast non-linear behaviour, in such a field the dynamic effects being unwanted parasitic phenomena. Under such an assumption, i.e.,  $T_A, T_B \ll T_0$ , the dynamic deviation  $e(t, \tau)$  is small for  $\tau \in [-T_A, T_B]$  even in the presence of full-range time evolutions of input  $s(t)$ . Therefore, Eq. (5) can be linearized with respect to  $e(t, \tau)$ :

$$y^{(D)}(t) = \int_{-T_A}^{T_B} w[s(t), \tau] e(t, \tau) d\tau + \Delta y^{(D)}(t) \quad ; \quad w = \left( \sum_{m=1}^{\infty} \frac{1}{m!} s^m(t) b_m^{(1)}(\tau) \right) \quad (6)$$

the linearization residual  $\Delta y^{(D)}(t)$  being negligible. The dynamic non-linear response  $y^{(D)}(t)$  is thus adequately described by means of a simple single-fold convolution, extended over the non-linear memory-time of the system, between the dynamic deviation at the input and a kernel  $w$ , which is non-linearly controlled by the instantaneous value  $s(t)$ . Neither “small-signal” operating conditions nor any hypotheses which can somehow limit the amplitude of the input have been assumed in the derivation of Eq. (6). Only the “short non-linear memory” hypothesis has been introduced. It is worth noticing that the dynamic linear contribution to  $y(t)$  in Eq. (4) is not at all affected by such an assumption, and linear dynamics described by a long-lasting pulse response  $h_1(\tau)$  is still fully supported by the present approach.

### III. The Discrete-Time Convolution Model for A/D channels

From a very general standpoint, either a stand-alone ADC or an entire digital data acquisition channel can be described as an ideal component which samples, discretizes and converts to digital at time instants  $t_k$  the output  $y(t)$  of a non-linear dynamic system (Fig. 1a). The latter describes, according to a behavioural approach, all the non-linearities (i.e., both static and dynamic) associated with the A/D channel. Since also in A/D channels the non-linear dynamics can be traced back to the “fast” non-linear electron devices used at the sampling and conversion sections, while the front-end architecture usually includes linear blocks with much longer memory (filters, attenuators/amplifiers, transmission lines), the non-linear system in Fig. 1a can be considered with “short” memory-time duration, and preceded by a purely-linear system with instead no particular constraints on its memory, whose input coincides with the input  $s_I(t)$  of the channel. Therefore, the non-linear block can be described by means of the modified Volterra series approach discussed in the previous Section, the overall model for the A/D channel becoming that functionally depicted in Fig. 1b, with

$$y^{(S)}(t) = z_0[s(t)] \quad ; \quad y^{(D)}(t) = \int_{-T_A}^{T_B} w[s(t), \tau] e(t, \tau) d\tau \quad ; \quad s(t) = \int_{-\infty}^{\infty} h(\tau) s_I(t - \tau) d\tau \quad (7)$$

The scheme in Fig. 1b clearly offers an inherent separation among different non-ideality effects in the response of the channel, which directly derives from the application of the dynamic deviation-based series approach. By assuming, without any loss of generality, a unitary DC response for the linear block,  $y^{(S)}$  represents the static response of the channel, or the response under slowly varying input signals (quasi-static operation), which can be characterized in terms of typical parameters such as offset, gain error, INL and DNL. The transfer function of the linear block can instead be experimentally extracted under frequency-dependent, small-amplitude sinusoidal excitations. Finally, the characterization of the purely-non-linear and purely-dynamic response  $y^{(D)}$  offers a robust starting point for the qualification of the non-linear dynamics of the A/D channel. To this aim, in order to obtain a finite set of operative parameters Eq. (6) can be rewritten by discretizing the convolution interval according to a time-step  $\Delta\tau$ , while the non-linear dependence of the residual first-order modified kernel  $w[s(t), \tau]$  with respect to  $s(t)$  can be re-expressed again in terms of a polynomial expansion, but now truncated to the finite order  $N$ :

$$y^{(D)}(t) = \sum_{\substack{p=-P_A \\ p \neq 0}}^{P_B} [s(t - p\Delta\tau) - s(t)] \sum_{n=1}^N \beta_{pn} s^n(t) + \Delta y^{(D)'}(t) + \Delta y^{(D)}(t) \quad (8)$$

The additional error  $\Delta y^{(D)'}(t)$  to Eq. (6), which is due to both the truncation of the polynomial and the discretization of the convolution interval, can be made adequately small by increasing  $N$  and reducing  $\Delta\tau$ , respectively. The functional description of Fig. 1b, when considering the operative formulation for  $y^{(D)}$  given in Eq. (8), represents the Discrete-Time Convolution Model formulation for the digital data acquisition channel.

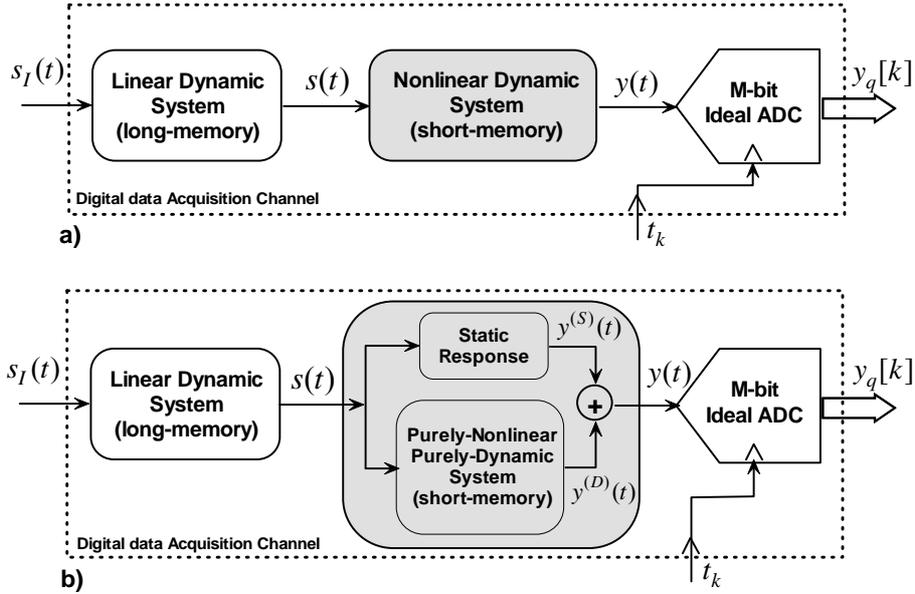


Figure 1. a): Behavioural block description for a digital data acquisition channel (also valid for a stand-alone ADC) and b): application of the DTCM approach to the system which takes into account non-linear effects.

#### IV. The DTCM parameters for the qualification of non-linear dynamics

The analytical discussion in Section II should have pointed out how the DTCM approach is not only a modelling strategy which can allow for both the accurate prediction of the A/D channel response to a given input signal and the adequate compensation of the distortion effects due to the channel non-idealities, but can also be considered, from a different standpoint, as a characterization procedure aimed at the parametric description of the purely-non-linear dynamics in the system behaviour. The DTCM parameter set  $(T_A, T_B, \Delta\tau, \boldsymbol{\beta} = \{\beta_{pn}\})$  (along with the related extraction procedures) provide an adequate starting point, with a robust theoretical justification, towards the definition of suitable metrics and dynamic figures of merit, for a possible standardization of ADC/acquisition channel testing and qualification in terms of dynamic non-linearities.

The correct choice of the non-linear memory-time duration around the instant  $t$  is a key factor for the accuracy of the overall method. This *a priori* parameter should be adequately small in order to satisfy the "short non-linear memory" hypothesis, but can not lie under the inferior limit which guarantees a negligible memory truncation error in Eq. (5). In the practice, however, such an inherent inferior threshold is always observed, since the actual limitation to further reductions of the memory-time is set by the experimental procedures required for the DTCM extraction, which need the estimation of the test input signals within  $[t_k - T_B, t_k + T_A]$ ,  $t_k$  being the generic sampling instant at which the channel is operated [6]. A too short memory-time can in fact introduce serious uncertainty issues in the capability of correctly reconstructing the required set of input signal values within such a reduced time interval. It is worth noticing that in most of the cases there are no substantial motivations for an asymmetric choice of the non-linear memory-time. Thus, in the following, the assumption  $T_A = T_B = T_M$  will be introduced, whereas in the general case a maximum memory duration  $T_M = \max\{T_A, T_B\}$  can be much rigorously considered. For a channel with input analogue bandwidth  $f_C$  and vertical range  $A_0$ , a worst-case discussion must assume the application of a  $A_0$ -amplitude sinusoidal signal of period  $T_0 = T_C = 1/f_C$ . This implies the choice  $T_M \ll T_0 = T_C$  for the non-linear memory duration. In Fig. 2a, the value of  $\max\{|e(t, \tau)|\}_{t, \tau}$  versus the ratio  $T_0/T_M$  is reported, for the specified input test signal  $s(t)$ . The vertical axis is normalized with respect to the input range  $A_0$ .

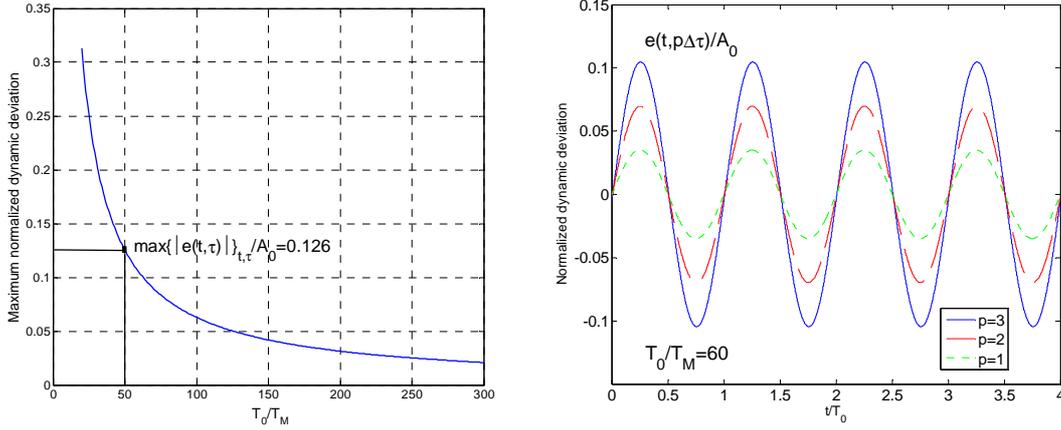


Figure 2. a) (left): Maximum value of the dynamic deviation function (normalized to the channel input range  $A_0$ ) versus the ratio  $T_0/T_M$ , for the worst-case sinusoidal input described in the text. b) (right): Time evolution of the  $\tau$ -discretized dynamic deviation ( $p=1,2,3$ ) ( $P=3$ ) for a sinusoidal input of amplitude  $A_0$  and period  $T_0$ .

The curve exhibits a hyperbolic-like evolution with  $T_0/T_M$ , this suggesting that a suitable trade-off choice for the ratio can be identified in proximity of the symmetry line. A reference value of  $T_C/T_M = 50$  is already reasonable in common applications for both setting a short non-linear memory-time duration with respect to the minimum input period  $T_C$  (thus guaranteeing small values of the dynamic deviation even in the worst-case) and preserving, at the same time, the feasibility of the DTCM extraction procedure. Clearly, with low-to-moderate input bandwidth systems the ratio can be chosen much higher, whereas ultra-fast ADCs or A/D channels can reduce the possible choices to lower values. As far as the discretization of the memory-time is concerned, it should be noticed that under the aforementioned hypotheses the dynamic deviation function exhibits a nearly linear evolution with  $\tau$ . For this reason, a quite raw discretization can be implemented, which allows for a moderate number of parameters within the matrix  $\beta$ , without introducing a relevant discretization contribution to  $\Delta y^{(D)}$  in (8). A value from 3 to 5 can be considered as an optimal trade-off for  $P = P_A = P_B$ . This also contributes to the accuracy of the model extraction procedures in the presence of very short memory-time durations. The value of the convolution discretization step  $\Delta\tau$  is thus accordingly derived. Figure 2b (normalized axes) shows the time evolution of the terms  $e(t, p\Delta\tau)$  of Eq. (8) (i.e., the  $\tau$ -discretized dynamic deviation) for a sinusoidal input of amplitude  $A_0$  and period  $T_0$ , with  $T_0/T_M = 60$  and  $P=3$ . The curves illustrate the small-valued, linear-like behaviour of  $e(t, \tau)$  in the  $\tau$ -domain, at each instant  $t$  during the application of a single-tone input signal near the upper bound of the channel bandwidth.

In order to preliminarily investigate the relationship between each DTCM parameter  $\beta_{pn}$  and the performance of the overall A/D channel in terms of dynamic non-linearity, the response  $y^{(D)}(t)$  in Eq. (8) can be rewritten as follows

$$y^{(D)}(t) = \sum_{\substack{p=-P_A \\ p \neq 0}}^{P_B} \sum_{n=1}^N y_{pn}^{(D)}(t) = \sum_{\substack{p=-P_A \\ p \neq 0}}^{P_B} \sum_{n=1}^N \beta_{pn} k_{n+1}(t, p\Delta\tau) \quad ; \quad k_m(t, \tau) = s^{m-1}(t)s(t-\tau) - s^m(t) \quad (m=1,2,\dots) \quad (9)$$

$k_m(t, \tau)$  being the  $m$ th-order *dynamic polynomial*. Equation (9) describes the non-linear purely-dynamic response of the channel as a linear combination with coefficients  $\beta$  of dynamic polynomials up to the  $(N+1)$ th-order, each evaluated at the corresponding discrete convolution-time instant  $\tau_p = p\Delta\tau$ . In Fig. 3, the spectra of the dynamic polynomials of Eq. (9) for  $p=3$  are shown with respect to the 0-dB level defined by the channel input range  $A_0$ , for a sinusoidal input signal of full-range amplitude  $A_0$  and period  $T_0$ , with  $T_0/T_M = 60$ ,  $P=3$  and  $N=7$ .

Besides the analytical investigation concerning the non-linear purely-dynamic elementary response of the system associated with the single  $\beta_{pn}$ , the definition of different DTCM-based figures for the A/D channel is being carried out taking also into account the harmonic partial cancellations deriving from the superposition of the  $\beta$ -weighted contributions  $k_{n+1}(t, p\Delta\tau)$  in Eq. (9). As an example, by considering again the sinusoidal input signal of amplitude  $A_0$  and period  $T_0$  with the choice  $T_0/T_M = 60$  and  $P=3$ , Fig. 4 shows the magnitude of the relative vector superposition of two spectral contributions  $Y_{pn}^{(D)}(f) = \mathfrak{F}\{y_{pn}^{(D)}(t)\}$  at

harmonic  $f_h = hf_0$  with different values either of index  $p$  (discretization) or  $n$  (order), versus the ratio of the corresponding weights  $\beta_{pn}$ . In particular, Fig. 4 illustrates the harmonic cancellation for different orders and same  $p$  (left), different convolution-time instants and same  $n$  (right), and for the interesting case of contributions  $y_{pn}^{(D)}(t)$  of the same order but corresponding to dynamic deviations evaluated at points in the  $\tau$ -domain which are symmetrical with respect to the origin (centre). The harmonic cancellations are never complete, but finite, narrow local minima can be identified in proximity of the unitary beta-ratio.

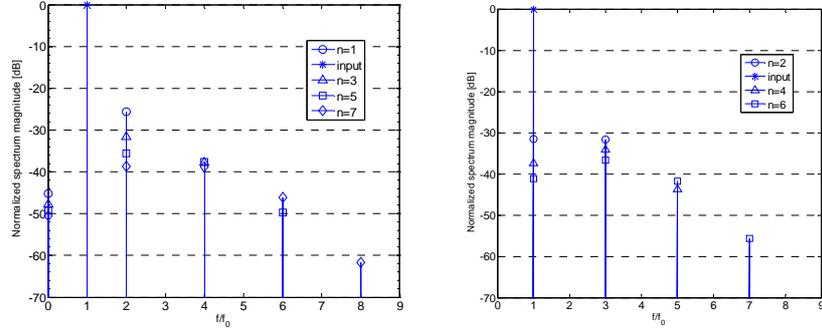


Figure 3. Spectra of the dynamic polynomials of Eq. (9) ( $p=3$ ), for a sinusoidal input signal of period  $T_0$  ( $T_0/T_M=60, P=3, N=7$ ).

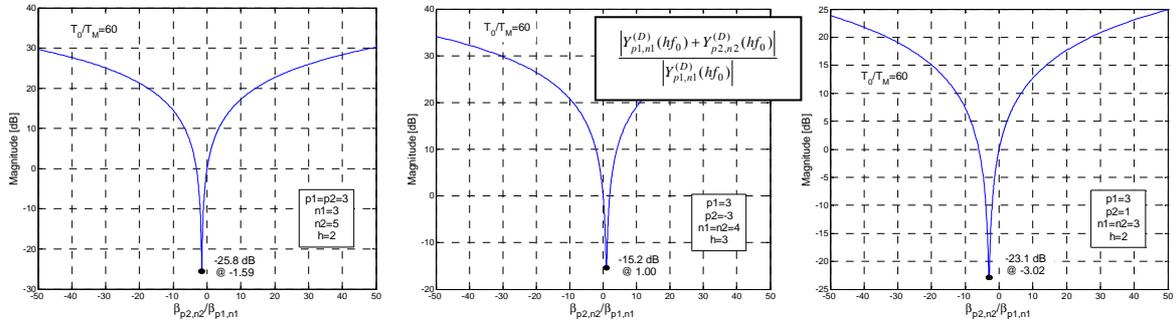


Figure 4. Harmonic partial cancellation at a given  $f_h = hf_0$  in the spectrum of the superposition of two contributions  $y_{pn}^{(D)}(t)$  (Eq. (9)) with different index  $n$  (left), index  $p$  (right) versus the beta-ratio, and for contributions of the same order and corresponding to points in the  $\tau$ -domain symmetrical w.r.t. the origin (centre) (sinusoidal input of period  $T_0$ ,  $T_0/T_M=60, P=3$ ).

## V. Conclusion

The properties of the Discrete-Time Convolution Model for A/D channels have been discussed from a different standpoint, by focusing on the analytical features of the dynamic deviation-based modified Volterra series from which it derives. In particular, the capability of the latter of separately describing the system non-idealities on the basis of their nature has been pointed out. This shows how the DTCM parameters are suitable for the in-progress definition of figures for the separate qualification of the purely-dynamic non-linearity in both stand-alone ADCs and digital data acquisition channels, aimed at supplementing the dynamic parameters of present standards.

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