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Figures of Merit for Analog-to-Digital Converters: The Optimal Set for the Uncertainty Evaluation

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Abstract-Choosing the right analog-to-digital converter for a stated measurement application and for a stated target uncertainty is not an easy task. In fact, each time the prospective purchaser browses among the manufacturer specifications, he finds different sets of parameters which qualify the product; moreover, the same figure of merit is often defined and measured in ambiguous way. One of the main reasons which has caused this situation is the coexistent of various Standards concerning the characterization of the analog-to-digital converters. The choice of the converter becomes more difficult taking into account that the potential buyer has to deal with the measurement uncertainty. Without question, the homogeneity of the figures of merit would facilitate the measurement evaluation. Target of the paper is the choice, among the large number of parameters proposed by the various Standards, of the optimal set of merit figures, which allows a correct uncertainty evaluation of a generic measurement performed by using an analog-to-digital converter. This want be a contribution toward the long awaited harmonization of the aforesaid Standards.

I. Introduction

Among the main applications of analog-to-digital converters (ADCs), there is doubtless their employment in the measurement world. There is, indeed, a huge variety of measurement instruments based on the analog-to-digital (A/D) conversion and the successive digital processing of the acquired data: beside the standalone instruments, usually dedicated to a defined measurement purpose, the measurement instruments approached as a data acquisition board connected to a common personal computer offer a more and more suitable and inexpensive alternative. In both cases, however, the choice of the suitable ADC, among the unlimited assortment of typologies and models, is not a straightforward operation; the ADCs manufacturers, in fact, declare different sets of parameters to characterize their products and, moreover, these parameters are often defined and measured in different ways. One of the reasons which has led to this situation [1] is the contemporaneous presence of various Standards concerning the characterization of the ADCs.

The task becomes more complicated considering that the instrument developers have to deal with the measurement uncertainty assessment. As prescribed by the “Guide to the Expression of Uncertainty in Measurement (GUM) [2], one of the crucial points in the uncertainty estimate is the standard uncertainty evaluation of each error source which gives a contribution to the measurement result. Any measurement instrument designer, which has to choose an ADC or a data acquisition board, would definitely take advantage of the homogeneity of the parameters declared in the specifications by the manufacturers of ADCs. During the uncertainty management stage, another advantage for the designers would be the chance to perform the uncertainty evaluation starting from a very limited set of figures of merit, in order to simplify the study of the combination and the propagation of the uncertainties.

In the last years, many Authors have dealt with the uncertainty evaluation in the measurement performed by using an ADC [3-6], proposing different approach to analyze the uncertainty propagation (Monte Carlo approach; propagation law of the GUM; random-fuzzy variables). We also handled the topic and proposed a numerical approach [7], based on the Monte Carlo method, to study the way the errors generated in the instrument hardware block propagate through the software block during the digital processing of the acquired data. In all these papers [3-7], it is clearly pointed out that starting from the manufacturer specifications (therefore performing a type B evaluation of the standard uncertainties associated with each error source) is the least expensive, the least time consuming and, often, the most accurate way to assess the uncertainties. In these cases, the uniformity, among the ADCs manufacturers, in declaring, defining and measuring the figures of merit which qualify their products is essential. This could be achieved only thanks to a full harmonization of all the Standards dealing with the characteristics of ADCs.

With the hope to give a contribution toward this harmonization, in this paper we propose and validate the usage of only five figures of merit to accurately estimate the uncertainty in a generic measurement based on the analog-to-digital conversion. The start basis of our proposal is the already mentioned numerical approach, which is briefly described in the next chapter.

II. The reference approach for the uncertainty evaluation

For the characterization of a measurement performed by using an ADC, according to [2], the first step to perform is the identification of the error sources which give a contribution to the uncertainty of the measurement result during the A/D conversion (in this framework, the errors generated during the digital signal processing are not considered, since these errors are usually negligible if the software block of the instrument is correctly designed). Obviously, the coexistence of various Standards dealing with the figures of merit which qualify the ADCs, does not help whoever has to handle the uncertainty management. In fact, the several Authors, which have proposed methodologies for the uncertainty assessment, use different figure of merit as starting point. The evaluation is performed by using: offset, gain and Effective Number Of Bits (ENOB) in [3]; offset, gain, non linearity, quantization and noise in [4]; offset, gain, quantization and noise in [5]; offset, gain and quantization in [6]. Our approach [7] starts from the specifications of offset (included its temperature drift and its long term stability), gain (included its temperature drift, its long term stability and uncertainty of onboard calibration reference), integral non-linearity (INL), spurious tones, thermal noise, settling time, timing jitter, quantization, differential non-linearity (DNL) and crosstalk.
In any case, after the identification of the uncertainty sources, to assess the combined standard uncertainty of the measurement results, other two steps have to be carried out: composition of the uncertainties associated with each source to obtain the combined uncertainty of each acquired sample; study of how the uncertainties of each acquired sample propagate during the digital signal processing.
To perform simultaneously these tasks we proposed a numerical approach based on the Monte Carlo method, developing a software tool which simulates a true A/D conversion and takes into account all the uncertainty sources. The tool is placed between a measurement input signal simulator and the software block which implements the measurement algorithm. It simulates a set of measurements carried out by using different realizations of the same ADC. In the following, its working principle is described. The input signal simulator generates N samples as if they were obtained from an ideal sampling process of the signal and the N samples are sent to the ADC simulation tool. The core of the tool is a FOR loop executed M times. The N samples vector, inside the loop, is modified in order to simulate the errors generated during the A/D conversion process. To simulate the offset, a constant value is added to each sample of the signal. This value is a random number within the range declared by the manufacturer. For each simulated measurement, the generated random number changes so that it lies in the specification range according to a rectangular distribution. In the same way, gain errors are simulated. In this case each sample of the signal is multiplied by a constant value. A white noise is added to simulate the thermal noise and, to simulate the crosstalk interference, another signal is added. The INL errors are simulated distorting the transfer function with components of second, third, fourth and fifth order so that the maximum deviation from a linear transfer function is always equal to the maximum INL value declared in the specifications. Two sinusoidal signals are added to simulate the presence of spurious tones. As for the settling time errors, the software tool calculates the errors for the actual sampling rate and the actual step between each two contiguous samples, starting from the full-scale settling time accuracy at the maximum sampling rate declared by the manufacturer. The timing jitter errors are simulated by multiplying a random number, within the range of aperture jitter declared in the specifications, by the derivative of the signal; the so obtained values, which are the amplitude errors caused by the sampling time errors, are added to each sample. At last, the simulation of the quantization process, which takes into account the DNL errors, is performed. The so modified N samples are sent to the software block of the measurement instrument, which calculates the measurement result. The M measures are collected outside the loop and the standard deviation of the measurements results, that is the combined standard uncertainty, is calculated.
The main advantage of this method is that it intrinsically takes into account every possible correlation between each quantity. However, it is obvious that the effectiveness of the described approach is strictly depending on how the A/D conversion process and the introduction of the errors are simulated. So with the aim of validating the approach, we applied the numerical method on various DSP basic blocks, which are typical of a measurement chain. The obtained results have been compared with the ones obtained by means of experimental tests and the comparison, as described in detail in [7], has positively validated the numerical method.

III. Definition of the optimal set and proposed approach for the uncertainty evaluation

Although the suggested approach leads to an accurate estimation of the uncertainties of a measurement performed by using a generic ADC, it is necessary to truthfully simulate a real A/D conversion process. This entails a large usage of resources and time both for developing the software and for performing the M simulations. Moreover, the method requires a high number of parameters, which not always can be found in the manufacturer specifications.

With the aim to overtake these limitations, we studied, by means of a frequency domain analysis, the effect of each error source on the data obtained by acquiring a single tone signal:

- the offset error reveal itself as a change of the DC component;
- the gain error produces a variation of the input signal spectral line amplitude;
- the INL causes the appearance of harmonics of the input signal;
- the spurious signals, which for various reasons can interfere with the A/D conversion process, appear as the corresponding frequency components.
- thermal noise, settling time, timing jitter, quantization and DNL generate a broadband noise, which, with good approximation, can be considered uniformly distributed all over the acquired spectrum;

Summarising, offset errors, gain errors, INL and spurious tones always operate on well defined spectral components, while on the contrary, the other uncertainty sources affect the whole frequency spectrum.

Another useful classification is the following:

- offset errors and spurious tones are input signal independent;
- gain errors and INL are input signal dependent;
- the broadband noise is actually input signal dependent, since some of the error sources, which generate it, depend on the shape, amplitude and frequency of the input signal (e.g. the timing jitter has no impact on a DC signal, but produces a evident effect on high frequency signals).

From these considerations, therefore, we can state that the merit figures, sufficient to perform an accurate measurement uncertainty evaluation, are: offset, gain; a parameter which quantifies the harmonic distortion; a parameter which quantifies the spurious tones; a parameter which measures what is commonly called *noise floor*.

Obviously, it would be very useful that these five parameters could be defined in an unambiguous way. Unfortunately, the Standards concerning the ADC characterisation do not apply a unified approach [1]. In particular we refer to the most recent and utilised publication:

- IEEE Std 1241, 2001, “Standard for Terminology and Test Methods for Analog-to-Digital Converters” [8]. This Standard is strongly based on the IEEE Std. 1057, 1994, “Standard for Digitizing Waveform Recorder” and provides both standard terminology for specifying the ADC performances and the test method for measuring it.
- DYNAD–SMT4-CT98-2214, “Methods and draft standards for the DYNamic characterization and testing of Analogue to Digital converters”, 2001 [9]. This Standard deals only with parameters that characterize the dynamic behaviour of ADCs and furnishes very detailed indications on how measuring these parameters.
- IEC 62008, “Performance characteristics and calibration methods for digital data acquisition systems and relevant software”, 2005 [10]. This Standard covers: the minimum specifications that the device manufacturer must provide to describe the performance of the A/D module; standard test strategies to verify the minimum set of specifications; the minimum calibration information required; the minimum calibration software requirements for external and self-calibration of the device.

At this point, let’s have a look on if and how the five figures of merit, proposed for the uncertainty estimation, are defined in the aforesaid Standards:

Offset and Gain. These figures are not defined in [9] since it deals only with dynamic parameters. In [10], offset is defined as the difference between the actual and ideal first transition level; gain is defined as the difference of actual and the ideal transition voltages in the transfer diagram at the specified gain point, after the offset has been adjusted to zero. In [8] there are two definition of offset and gain: (A - independently based definition) gain and offset are the values by which the input values are multiplied and then to which the input values are added, respectively, to minimise the mean squared deviation from the output values; (B - terminal-based definition) gain and offset are the values by which the input values are multiplied and then to which the input values are added, respectively, to cause the deviations from the output values to be zero at the terminal points, that is, at the first and last codes. Despite the fact that there are noticeable differences in the various definitions of offset and gain, it is possible to affirm that these differences lead to negligible disagreement in the measurement of these parameters.

Harmonic Distortion. In the three considered Standards, the harmonic distortion is quantified by the same parameter called *Total Harmonic Distortion* (THD). IEC 62008 states that THD is, for a sine wave signal, the sum of power of all harmonics. In [8] THD is defined as, for a pure sinewave input of specified amplitude and frequency, the root-sum-of-squares (rss) of all the harmonic distortion components including their aliases in the spectral output of the ADC. Unless otherwise specified, THD is estimated by the rss of the second through the tenth harmonics, inclusive. THD is often expressed as a decibel ratio with respect to the root-mean-square amplitude of the output component at the input frequency. In DYNAD, THD is, for a pure sinewave input of specified amplitude and frequency, the ratio of the rss of all the harmonic distortion components, including their aliases in the spectral output of the ADC, to the rms amplitude of the output component at the input frequency, expressed in dB. Unless otherwise specified, THD is estimated by the rss of the second through the tenth harmonics, inclusive. In the three Standards, therefore, the THD parameter quantifies the same phenomenon. Obviously, since the proposed definitions are different, this leads to different values of the same figure of merit. For our purpose, in the following we refer to the DYNAD definition.

Spurious Tones. The spurious tones are quantified by a parameter called *Total Spurious Distortion* (TSD). In [8] TSD is defined as, for a pure sinewave input of specified amplitude and frequency, the root-sum-square of the spurious components in the spectral output of the ADC. TSD is often expressed as a decibel ratio with respect to the root-mean-square amplitude of the output component at the input frequency. In [9] TSD is defined as the root of the sum of the powers of the spurious components in the range from DC (excluded) up to half the sampling frequency, expressed as a dB ratio to the rms amplitude of the output component at the input frequency, for a pure sinewave of specified amplitude and frequency stimulus. In [10] there are not parameters which directly quantify the presence of spurious tones. Also in this case, in the following we refer to the DYNAD definition.

Noise Floor. Rather more complicated is to find a figure of merit which characterise the noise floor. This could be the *Signal to Non-Harmonic Ratio* (SNHR). In [8], SNHR is defined as, for a pure sine-wave input of specified amplitude and frequency, the ratio of the rms amplitude of the ADC output signal to the rms amplitude of the output noise which is not harmonic distortion. In [10] the SNHR is quite different, since it is expressed as ratio, expressed in dB of the power of the signal with all possible harmonics, to the overall noise. In both cases, however, the spurious components are included in the output noise; therefore this parameter cannot be used for the uncertainty estimation. In [9] SNHR is not even mentioned and in order to quantify the noise floor, is defined the *Signal-to-noise ratio* (SNR) is introduced: SNR is defined as a measure of the broadband noise that is introduced into the output signal by the sampling and analogue-to-digital conversion processes. It is given by the ratio expressed in dB of the rms amplitude of the ADC output fundamental tone to the rms amplitude of the output noise, where noise is defined as the sum of all frequencies different from the fundamental, harmonics, and spurious components in the range from DC (excluded) up to half the sampling frequency. However, the same DYNAD suggests to not encourage the usage of this figure of merit, in IEEE 1241, the SNR is considered ambiguous and in [10] SNR is not even mentioned. In order to avoid this confused situation, it could be useful to define, in unambiguous way, a parameter which takes into account only the broadband noise, defined in the same way the SNR is defined in [9]. This figure of merit could be called *Signal to Noise Floor Ratio* (SNFR).

From the analysis made above, we can state that, starting from the merit figures of offset, gain, THD, TSD and SNFR, it is possible to perform the uncertainty evaluation of whatever measurement performed by using an ADC.

In order to prove this statement, we applied the Monte Carlo approach and developed a simple simulator of the A/D conversion. The simulation of the errors represented by the selected parameters is performed in the following way:

- to simulate the offset errors, a constant value is added to each sample of the input signal. This value is a random number within the range declared by the manufacturer. For each trial, the generated random number changes so that it lies in the specification range according to a rectangular distribution;
- to simulate the gain errors, each sample of the signal is multiplied constant value. This value is a random number within the range declared by the manufacturer. For each trial, the generated random number changes so that it lies in the specification range according to a rectangular distribution;

- to simulate the THD errors, the transfer function is distorted with components from the second to the tenth order. The amplitude of these components, for each trial, produces an actual THD randomly distributed from 0 to the THD declared in the specification;
- two sinusoidal signals are added to simulate the presence of spurious tones. The amplitude of these components, for each trial, produces an actual TSD randomly distributed from 0 to the TSD declared in the specification;
- to simulate the noise floor errors, a gaussian noise equivalent to the SNFR is added to the input signal.

If compared to the one proposed in [7], the advantage of this method is, besides the reduction of the parameters used for the uncertainty evaluation, a huge reduction of the software tool complexity and of its execution time; in fact by using this version, the simulation of settling time, timing jitter and quantization (plus DNL) is performed just adding an equivalent noise to the input signal; obviously, since the SNFR is usually measured for the worst case (that is maximum conversion rate and maximum frequency of the input signal), this approach leads to a slight overestimate of the uncertainties.
It is to remark that THD, TSD and SNFR values, besides from the manufacturer specifications, can be obtained by a Type A evaluation, if an adequate sinewave generator is available. In this case, it is possible to refer to the actual values instead of using the worst case declared by the manufacturers and more accurate combined uncertainty values can be assessed.
By a Type A evaluation, moreover, is possible to take into account the effect of electromagnetic disturbances, which could produce other spurious tones and/or an increasing of the noise floor, and to take into account the actual crosstalk if a multi-channel acquisition is performed.

IV. Validation of the proposed approach

With the aim to validate the proposed approach, we applied it in various practical cases and compared the obtained results with the ones obtained by using the reference approach.
For instance, in table I are reported typical specifications for 12 bit and 16 bit ADCs with ± 10 V input range and working at a 10 KS/s rate (crosstalk is not included considering a single channel acquisition).

Table I - Typical specifications for 12 bit and 16 bit ADCs.

Specification	**offset**	**gain**	**INL**	**DNL**	**thermal noise**	**settling time for full scale step**	**time jitter**	**TSD**
12 bit ADC	± 1000 μV	± 0.05 %	± 1 LSB	± 0.5 LSB	0.07 LSB rms	± 0.1 LSB in 100 μs	± 5 ps	74 dB
16 bit ADC	± 150 μV	± 30 ppm	± 1 LSB	± 1 LSB	0.30 LSB rms	± 0.5 LSB in 100 μs	± 5 ps	94 dB

Putting as input for the reference simulator described in chapter II, a full scale 4900 Hz sinewave, we obtained the following values for the proposed figures of merit. For the 12 bit ADC: THD = 72 dB; SNFR = 71 dB. For the 16 bit ADC: THD = 96 dB; SNFR = 92 dB.
Starting from these values, let's apply both the reference approach and the proposed approach to the measurement of the DC value, the RMS value, the amplitude of a single tone by means of Fast Fourier Transform (FFT) and the THD %.
The input signal is a 555 Hz triangular waveform with a 8 V peak value plus a 1 V DC signal. The time window is 1 s.
In table II the results obtained by the application of both methods (performing 10000 trials) are reported.
Analyzing the results, it is possible to notice that the proposed approach leads to uncertainty estimations slightly greater than the ones obtained by using the reference methodology. The reason of this overestimate is that, with the proposed approach, we always add a noise equivalent to the worst case SNFR. With the reference approach, conversely, the truly simulation of the A/D conversion allows the correct evaluation of the effects of quantization, DNL, settling time and time jitter. However, it must be underlined that the application of the proposed approach requires a negligible amount of time and of resources, if compared with the time and the resources involved by the method suggested in [7].
We obtained similar results considering other measurement algorithms and other input signals.
In all cases, the results show that the usage of offset, gain, THD, TSD and SNFR is the right choice for the uncertainty evaluation of a generic measurement performed by means of an ADC.

Table II - Expected values and combined standard uncertainties.

	DC value	**RMS value**	**FFT**	**THD %**
Expected values	1.000 V	4.726 V	4.585 V	12.03
Uncertainties obtained by using the reference approach - 12 bit ADC	0.8 mV	1.2 mV	1.4 mV	$540 \cdot 10^{-6}$
Uncertainties obtained by using the proposed approach - 12 bit ADC	0.9 mV	1.3 mV	1.4 mV	$550 \cdot 10^{-6}$
Uncertainties obtained by using the reference approach - 16 bit ADC	95 μV	83 μV	81 μV	$48 \cdot 10^{-6}$
Uncertainties obtained by using the proposed approach - 16 bit ADC	98 μV	88 μV	82 μV	$49 \cdot 10^{-6}$

V. Conclusions

In this paper the problem of the uncertainty estimation of the measurements performed by using an ADC has been considered. According to the GUM, the various error sources, introduced during the A/D conversion, have been taken into account, and a numerical method has been applied to obtain the values of the combined standard uncertainty of the measurement results. By analysing the results of these simulations we found out that the usage of only five figures of merit, namely offset, gain, THD, TSD and SNFR, allows a correct evaluation of the measurement uncertainties. Obviously, our proposal does not diminish the significance of other figures of merit which can give useful information about ADCs, such as *Spurious-Free Dynamic Range* (SFDR), *SIgnal-to-Noise And Distortion ratio* (SINAD), *Effective Number Of Bits* (ENOB). Even if in this paper the five figures of merit have been used for a Monte Carlo approach, they could be safely employed as basis for other uncertainty propagation methodologies, such as the propagation law of the GUM or the random-fuzzy variables technique. The usage of the proposed parameters can be easily extended to more complex measurement chain, for instance when transducers and/or signal conditioning accessories (transformers, voltage probes, filters, amplifiers and so on) are connected to the ADC. In these case the offset, gain, THD, TSD and SNFR values shall be obtained for the whole measurement chain. Unfortunately, the main Standards concerning the ADC characterisation do not define offset, gain, THD, TSD and SNFR in an unambiguous way. However, in the recent years, many efforts have been made toward the harmonisation of these Standards and toward the release of a unified version. Our suggestion is that the proposed figures of merit should be included and defined in this awaited release.

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