13th Workshop on ADC Modelling and Testing
Sep. 22-24, 2008, Florence, Italy

Easy Estimation of Spectral Purity of Test Signals for ADC Testing

David Slepička

Czech Technical University in Prague, Faculty of Electrical Engineering, Dept. of Measurement
Technická 2, CZ-16627 Prague 6, Czech Republic
Phone: +420 2 2435 2058, Fax: +420 2 3333 9929, E-mail: slepicd@fel.cvut.cz

Abstract – ADC testing frequently confronts the problem of spectral purity of harmonic test signal. The majority of dynamic ADC test methods require test signal distortion at least 10 dB less than that of the tested ADC – the condition that cannot be practically fulfilled for testing up-to-date ADCs. More sophisticated approach to ADC testing applies the correction for test signal imperfections. In both cases test signal distortion has to be known – roughly in the first case and quite accurately in the second case. In this paper an easy method for the testing of signal spectral purity is proposed. Test signal passed by one of two simple analog filters is measured by a common ADC, whose nonlinearity is mathematically post-corrected. The results are compared with common approach applying notch filter and uncertainty analysis is performed.

I. Introduction

ADC dynamic testing is generally based on spectrally pure sine wave stimulus. Standards of ADC testing [1], [2] assume high-quality test signal and do not consider other case; or they eventually recommend test signal filtering by pass-band filters, which also leads to spectrally pure sine wave. Unfortunately the manufacture of such filters is neither easy nor cheap. Alternative non-standardized methods mostly assume the same test signal purity although it is hardly achievable in practice.
There are in principle three approaches to this problem:

- A signal produced by common generators is filtered by high-quality pass-band filters.
- The problem is restricted to harmonic distortion, multitone signal is used and intermodulation components are observed.
- Alternative easily-to-generate test signals are applied.
- Special methods enabling the correction for test signal imperfections are applied.

Test signal filtering is in principle an easy way how to get rid of spectral impurity but it is apparently the most difficult one in practice particularly at lower frequencies. The limiting factor is an extreme demand on components' linearity that leads to high mechanical dimensions at lower frequencies (audio band). Moreover, there remains a problem with phase noise that is difficult to be filtered out [3]. Thus special signal generators with successive filters have to be designed [4]. Disadvantage of such approaches is the limitation to one frequency at which the generator is tuned and high costs of generator's components.
The application of multitone signal suppresses the need of low harmonic distortion of the test sine wave. If the power combiner of all sine waves is linear enough, it is only the ADC nonlinearity producing intermodulation components in the acquired signal spectrum. The first three harmonic components dominate at the most ADCs; in this case the total harmonic distortion can be estimated from the intermodulation distortion [5]. It is also possible to assess the ADC transfer function from intermodulation components [6]. However, only ADC nonlinearity can be covered by this test and neither spurious components nor wideband noise. The noise of multitone signal cannot also be too high in order not to mask intermodulation components.
Another approach to minimize the test signal distortion is to use special test signals that could be generated with lower distortion than a common sine wave – e.g. exponential [7], Gaussian noise [8] or small triangles [9]. The common disadvantage of such approaches is that these signals contain many frequencies and the results of these tests are difficultly transferable to standardized test results.
The last group of methods mentioned applies common sine wave passed through a set of filters to the tested ADC and computes the actual ADC performance from several measurements [10], [11], [12]. This approach can be applied for the estimation both of harmonic distortion and of wideband noise of the tested ADC as shown in [12]. Limitations of these methods are the need of set of filters [10] or the level of ADC and test signal distortion that should not be far away [11], [12]. Another problem arises in vicinity of the fundamental where the accuracy of noise estimation is low [12].
In several publications (e.g. [10]), the possibility of correction for test signal imperfections has been

analysed. If the vector representation of test signal harmonic distortion is known, this is a simple task of complex subtraction. In some cases when the test signal distortion is small enough, this correction is not needed at all. So, the problem can be reduced on how to estimate test signal imperfections. In this paper a modified method based on simple passive filters and mathematical post correction [11], [12] is analysed, practical results are shown and uncertainties are estimated.

II. Proposed approach

The principle of the proposed method comes from the approach [11], [12] and the same measurement setup (see Fig. 1) slightly modified for the test signal measurement. Test signal $G(j\omega)$ is measured twice – each time passed through one of the filters $F(j\omega)$ or $H(j\omega)$. For the verification of results there is also a notch filter $N(j\omega)$ tuned at fundamental frequency of the test signal.

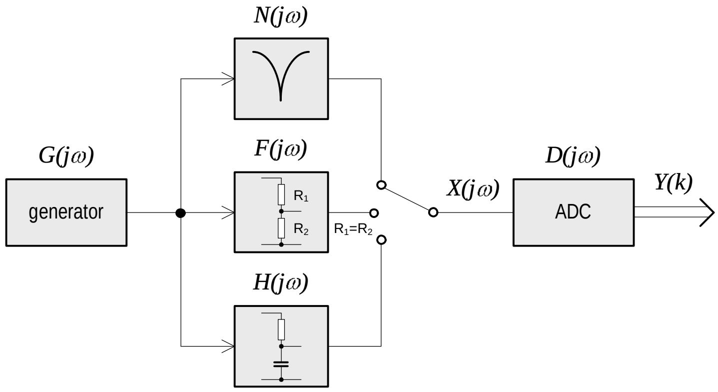


Figure 1. Block diagram of the measurement setup

Filter characteristics $F(j\omega)$ and $H(j\omega)$ can be almost random, there are only three requirements on them. They must be different, their attenuation at fundamental frequency must be the same and they must be linear so that they do not produce harmonic distortion.
The measurement of generator frequency spectrum consists of two parts: harmonic components and noise. This comes from the character of generator and ADC distortion. In case of (additive) noise, both generator and ADC frequency spectra are supposed to be independent (uncorrelated); thus output noise power consists of the sum of ADC input signal noise power and ADC noise power:

$$\mathrm{E}\left\{\left|Y_F(jw)\right|^2\right\}=\left|F(jw)\right|^2\mathrm{E}\left\{\left|G(jw)\right|^2\right\}+\left|D(jw)\right|^2,\quad \mathrm{E}\left\{\left|Y_H(jw)\right|^2\right\}=\left|H(jw)\right|^2\mathrm{E}\left\{\left|G(jw)\right|^2\right\}+\left|D(jw)\right|^2 \qquad (1),(2)$$

where $Y_1(j\omega)$[1] is the frequency spectrum measured when filter $F(j\omega)$ is applied and $Y_2(j\omega)$ measure with filter $H(j\omega)$. Generator noise is done by the formula

$$\mathrm{E}\left\{\left|G(jw)\right|^2\right\}=\frac{\mathrm{E}\left\{\left|Y_F(jw)\right|^2\right\}-\mathrm{E}\left\{\left|Y_H(jw)\right|^2\right\}}{\left|F(jw)\right|^2-\left|H(jw)\right|^2}. \qquad (3)$$

Harmonic distortion is a correlated distortion and frequency spectral lines have to be treated as vectors:

$$Y_F(jw)=F(jw)G(jw)+D(jw),\quad Y_H(jw)=H(jw)G(jw)+D(jw) \qquad (4),(5)$$

Generator harmonic distortion can be expressed as

$$G(jw)=\frac{Y_F(jw)-Y_H(jw)}{F(jw)-H(jw)}. \qquad (6)$$

For the computation of (6) filter characteristics have to be known. Frequency characteristic $F(j\omega)$ of the filter used in the measurement setup according to Fig. 1 is frequency independent and done by

$$F(jw)=F=\frac{R_2}{R_1+R_2}=\frac{1}{2} \qquad (7)$$

because $R_1 = R_2 = R$. If frequency characteristic $H(j\omega)$ is determined from components' characteristics,

[1] Continuous frequency ω is used as the argument of discrete frequency spectrum Y for the simplicity.

parasitic components are not incorporated and results could be biased. So, it is advantageous to determine this characteristic by measurement. An easy way is to use saw signal $T(j\omega)$, which contains odd and even harmonic components, instead of sine wave $G(j\omega)$ in Fig. 1 and compute $H(j\omega)$ from measurements $Y_{Ft}(j\omega)$ with filter $F(j\omega)$ and $Y_{Ht}(j\omega)$ with filter $H(j\omega)$ as

$$H(jw) = \frac{|Y_{Ht}(jw)|}{|T(jw)|} e^{j(f_{Ht}-f_T)} = \frac{|Y_{Ht}(jw)||F(jw)|}{|Y_{Ft}(jw)|} e^{j(f_{Ht}-f_{Ft}+f_F)} = \frac{F|Y_{Ht}(jw)|}{|Y_{Ft}(jw)|} e^{j(f_{Ht}-f_{Ft})}. \quad (8)$$

III. Measurement uncertainties

In this section the uncertainty of the generator harmonic distortion determined by (6) is analysed. As the source of uncertainties the quantization noise or any wideband noise is assumed.
The uncertainty of type B of (6) u_G is done by

$$u_G^2 = \frac{1}{(F-H)^2}\left(u_{Y_F}^2 + u_{Y_H}^2\right) + \frac{(Y_F - Y_H)^2}{(F-H)^4}\left(u_F^2 + u_H^2\right) \quad (9)$$

where indices of u sign appropriate uncertainties[2]. Since all components in (9) are in the complex form, the uncertainties are in the same form. General spectral component Y and its uncertainty are given by

$$Y = |Y| e^{jf} = \sqrt{2} Y_{rms} e^{jf}, \quad (10)$$

$$u_Y^2 = e^{2jf} u_{|Y|}^2 + \left(|Y| j e^{jf}\right)^2 u_f^2 = e^{2jf}\left(u_{|Y|}^2 - |Y|^2 u_f^2\right) = Y^2\left(|Y|^{-2} u_{|Y|}^2 - u_f^2\right) = Y^2\left(Y_{rms}^{-2} u_{Y_{rms}}^2 - u_f^2\right). \quad (11)$$

Formula (11) decomposes complex uncertainty into rms value and phase uncertainty [13]

$$u_{Y_{rms}}^2 = \frac{ENBW_0}{N} u_q^2, \quad u_{f(k)}^2 = \frac{NNPG}{2M^2(k)N} u_q^2 \quad (12), (13)$$

where $ENBW_0$ is equivalent-noise bandwidth of the window $w(n)$ (the usage of time window for leakage suppression is assumed) and given by [14], N is number of samples, u_q is quantization uncertainty and $NNPG$ is the normalized noise power gain of the used window $w(n)$:

$$ENBW_0 = N \frac{\sum_{n=0}^{N-1} w^4(n)}{\left(\sum_{n=0}^{N-1} w^2(n)\right)^2}, \quad u_q = \frac{U_{FS}}{2^{ENOB}\sqrt{12}}, \quad NNPG = \frac{1}{N}\sum_{n=0}^{N-1} w^2(n) \quad (14), (15), (16)$$

where U_{FS} is full-scale range and $ENOB$ is the effective number of bits. $M(k)$ is the module of amplitude frequency spectrum at frequency bin k. Unlike rms value uncertainty, phase uncertainty is the function of frequency (the frequency index is often omitted in this paper for the sake of simplicity).
Uncertainties u_{Y_F} and u_{Y_H} could be both determined applying (11). Uncertainty of F follows from (7)

$$u_F^2 = \frac{R_2^2}{(R_1+R_2)^4} u_{R_1}^2 + \frac{R_1^2}{(R_1+R_2)^4} u_{R_2}^2 = \frac{1}{8R^2} u_R^2, \quad u_R = \frac{\Delta_R}{\sqrt{3}} \quad (17), (18)$$

where u_R is the uncertainty of both R_1 and R_2 and done by their tolerance Δ_R. Uncertainty of H

$$u_H^2 = H^2\left(|H|^{-2} u_{|H|}^2 - u_{f_{Ht}}^2 - u_{f_{Ft}}^2\right), \quad |H| = F|Y_{Ht}||Y_{Ft}|^{-1} \quad (19), (20)$$

comes from (8) using (11). Uncertainty $u_{|H|}$ can be determined from (20) as

$$u_{|H|}^2 = \frac{F^2}{|Y_{Ft}|^2} u_{|Y_{Ht}|}^2 + \frac{F^2 |Y_{Ht}|^2}{|Y_{Ft}|^4} u_{|Y_{Ft}|}^2 + \frac{|Y_{Ht}|^2}{|Y_{Ft}|^2} u_F^2 = |H|^2\left(\frac{u_{|Y_{Ht}|}^2}{|Y_{Ht}|^2} + \frac{u_{|Y_{Ft}|}^2}{|Y_{Ft}|^2} + \frac{u_F^2}{F^2}\right). \quad (21)$$

It follows from (12) that module uncertainty is independent of frequency; thus $u_{|Y_{Ht}|} = u_{|Y_{Ft}|} = u_{|Y|}$.
Substituting (21), (17) and (7) into (19), uncertainty of H is

[2] All arguments $(j\omega)$ were omitted in the rest of this paper for the sake of notation simplicity.

$$u_H^2 = H^2\left(\left(\left|Y_{Ht}\right|^{-2} + \left|Y_{Ft}\right|^{-2}\right)u_{|Y|}^2 + \tfrac{1}{2}R^{-2}u_R^2 - u_{f_{Ht}}^2 - u_{f_{Ft}}^2\right). \tag{22}$$

Similarly as in (11), modules or rms values can be used in (22).

IV. Experimental results

Practical applicability of the proposed method was verified by experimental measurements on high-quality instruments. As signal source, ultra-low distortion generator Stanford Research DS360 at the signal frequency of 20.9 kHz was applied and as ADC, high-quality NI PXI-5922 digitizer, which has the highest dynamic range of any digitizer on the market, was used at the sampling frequency of 50 kHz. Filters ***F*** and ***H*** were manufactured using common metal resistors 3.9 kΩ with 1% accuracy and high-voltage foil capacitor 3.3 nF/650 V that proved to be sufficiently linear.
Measurements and computations were performed as described in section II. In each measurement 1 Msa data record was acquired, divided into 63 segments with 50% overlapping, Blackmann-Harris 7 term window applied to each segment and amplitude frequency spectrum was computed applying Welch method of averaging. Rms values of harmonic components were computed from the spectra according to well-known formula [15] and relative phases of higher harmonic components were estimated using a simple method [16]. For the verification of the proposed method, generators' frequency spectrum was measured through notch filter that decreased signal dynamic range. Notch filter frequency characteristic was also measured and used for the correction of the measured frequency spectrum $\boldsymbol{Y_N}$; thus, the influence of notch filter on measured results is minimal.
Results are shown in Fig. 2 and Tab. 1. From the spectrum of $\boldsymbol{Y_F}$ it is not clear if harmonic distortion and noise come from the generator or digitizer because their performance is roughly comparable. The correction revealed that test signal harmonic components dominate only up to 4th and its wideband noise is below ADC's one. Note that signal and noise levels are shifted up in Fig. 2b, c because they are expressed in dBfs units and they correspond to signal level before filtering (and attenuating).
The results of correction are in agreement with the results $\boldsymbol{Y_N}$ measured using notch filter. Only the second harmonic component was estimated with low accuracy. The reason was dc offset that was slightly different in each measurement and consequently the signal occupied slightly different part of ADC transfer function. Another source of this error was lower difference in filters' characteristics and consequently higher uncertainty (9). This also led to increased noise in the vicinity of the fundamental in the corrected amplitude frequency spectrum. The estimate of some higher particularly even harmonic components was also biased because of high noise level relatively to their level.
The accuracy of results was estimated by computing uncertainties according to section III. The ***ENOB*** was estimated from the ***SNHR*** (Signal to non-harmonic ratio) [1] of signal $\boldsymbol{Y_F}$ (Fig. 2a) as

$$ENOB = \frac{SNHR(\mathrm{dB}) - 1.76}{6.02} \tag{23}$$

because noise spectral density approaches to uniform. Uncertainty $\boldsymbol{u_G}$ was computed in the complex form, the range of modules of generator's harmonic components was computed with the coverage factor of 2 and phase expanded uncertainty with the same coverage factor was determined (see Tab. 2). Assuming values of corrected $\boldsymbol{Y_N}$ correspond to generator's characteristics, the computed values of ***G*** with expanded uncertainties are in agreement with the correct generator's values. The only errors appear at 2nd harmonic component, which was already discussed, and some weak harmonic components close to noise floor.

V. Conclusion

In this paper an easy-to-implement method for the assessment of harmonic signal's quality was proposed. It is based on several measurements of this signal passed through simple passive filters and posterior computation of generator's characteristics. The uncertainties of this method were determined for the practical case of incoherent sampling. Practical measurement and their verification proved the applicability of the proposed approach and computed uncertainties of measurements were in agreement with the real values. However, not only wideband noise, from which the uncertainties were determined, is a significant source of uncertainties but also instabilities and e.g. dc shift, which caused an increased error in experimental measurements. Practical applicability of this method is obvious – for fast and easy estimation of harmonic distortion and noise of generators but also of ADCs as proposed in [12]. The advantage of this approach is also that harmonic distortion is expressed in the complex form and consequently can be used for the correction of generator nonlinearity, too.

References

[1] "IEEE Std. 1241–2000: IEEE Standard for Terminology and Test Methods for Analog-to-Digital Converters", *The Institute of Electrical and Electronics Engineers*, Inc., New York, 2000.

[2] "DYNAD: Methods and draft standards for the dynamic characterization and testing of Analogue to Digital converters", *European project SMTC-CT98-2214,* http://www.fe.up.pt/~hsm/dynad, 2001.

[3] M. Komárek, V. Papež, J. Roztočil, P. Suchánek, "Sine-Wave Signal Sources for Dynamic Testing High-Resolution High-Speed ADCs", *XVIII-th IMEKO World Congress,* CD-ROM, 2006.

[4] V. Papež, S. Papežová, "Highly Pure Sine-Wave Signal Sources for ADC Testing", *15th IMEKO TC 4 International Symposium,* Iasi, vol. 1, s. 35–40, 2007.

[5] M. Benkais, S. Masson, P. Marchegay, "A/D Converter Characterization by Spectral Analysis in "Dual-Tone" Mode", *IEEE Transactions on Instrumentation and Measurement*, vol. 44, no. 5, pp. 940–944, 1995.

[6] D. Slepička, D. Dallet, V. Shitikov, F. Barbara: "ADC Characterization in the Frequency Domain by Dual Tone Testing", *XVIII-th IMEKO World Congress,* CD-ROM, 2006.

[7] R. Holčer, L. Michaeli: "Testing DNL and INL of ADC by the exponential shaped voltage", *ADDA & EWADC 2002,* pp. 99–102, 2002.

[8] J. Holub, J. Vedral, J. Kubín: "Improvement of Stepp-Gauss ADC Stochastic Test Method", *ADDA & EWADC 2002,* pp. 115–118, 2002.

[9] F. Alegria, P. Arpaia, A. C. Serra, P. Daponte: "Performance analysis of an ADC histogram test using small triangular waves", *IEEE Transactions on Instrumentation and Measurement*, vol. 51, no. 4, pp. 723–729, 2002.

[10] V. Haasz, D. Slepička, "Frequency Spectrum Correction Method for the ADC Testing", *IEEE IMTC 2004*, pp. 533–536, 2004.

[11] Y. Zhongjun, C. Degang; R. Geiger, "Accurate testing of ADC's spectral performance using imprecise sinusoidal excitations", *2004 International Symposium on Circuits and Systems ISCAS '04*, vol. 1, pp. 645–8, 2004.

[12] D. Slepička, D. Dallet, V. Shitikov, F. Barbara, "Posteriori Frequency Spectrum Correction for Test Signal Imperfections in ADC Testing", *IEEE IMTC 2007*, CD-ROM, 2007.

[13] M. Novotný, D. Slepička: "Uncertainty Analysis of the Phase and RMS Value by Non-Coherent Sampling in the Frequency Domain," *IEEE IMTC 2005,* pp. 2114–2117, 2005.

[14] P. Carbone, E. Nunzi, D. Petri: "Windows for ADC dynamic testing via frequency-domain analysis", *IEEE Transactions on Instrumentation and Measurement*, vol. 50, no. 6, pp. 1571–1575, 2001.

[15] O. M. Salomon: "The Use of DFT Windows on SNR and Harmonic Distortion Computations," *IEEE Transactions on Instrumentation and Measurement*, vol. 43, no.2, pp 194–199, 1994.

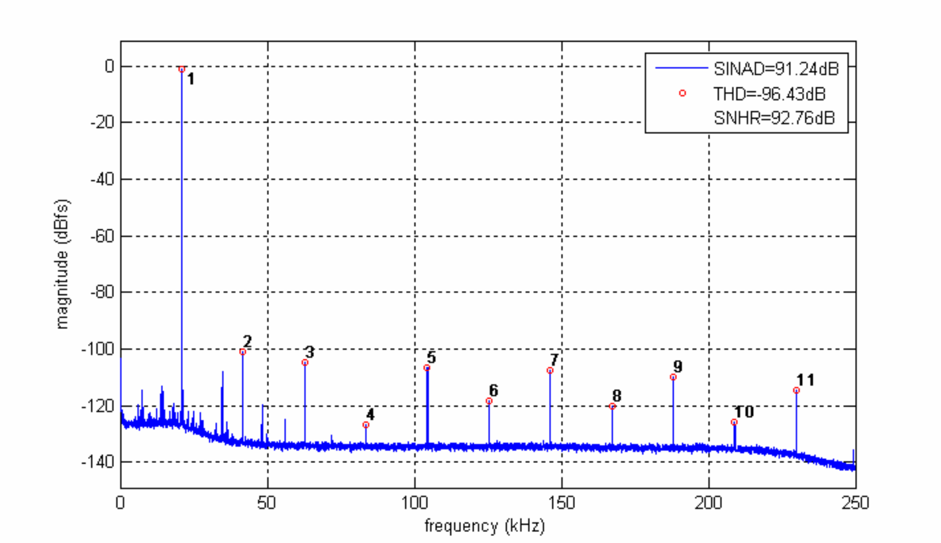
[16] D. Slepička: "Phase Differences Determination between the Fundamental and Higher Harmonic Components at Non-coherent Sampling," *13th IMEKO TC-4 Symposium*, pp. 242–247, 2004.

Table 1. Result comparison

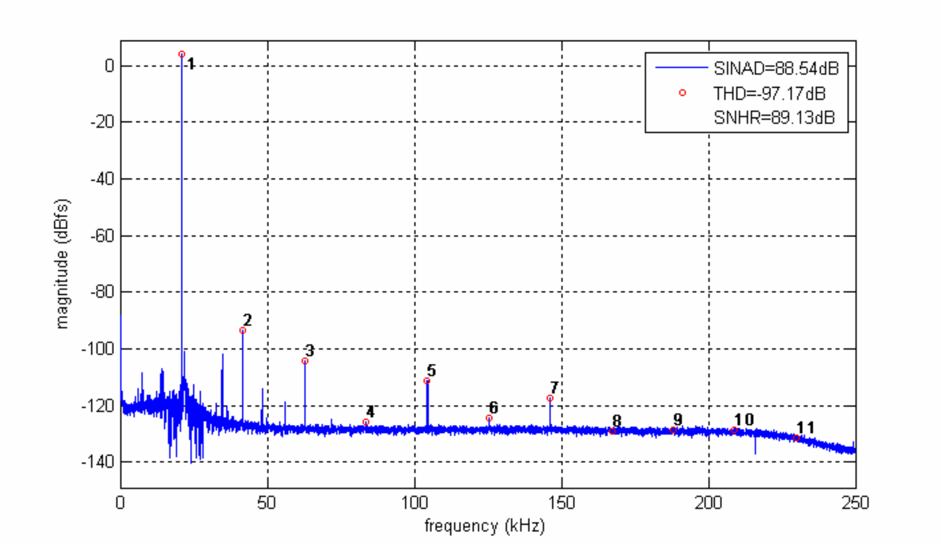
		harmonic component									
signal	parameter	2nd	3rd	4th	5th	6th	7th	8th	9th	10th	11th
Y_F	M (dBc)	–99.7	–103.6	–123.1	–105.0	–116.9	–106.6	–118.5	–108.6	–123.0	–113.2
	Δφ (rad)	0.58	2.48	–0.57	–2.85	0.01	2.96	0.08	–3.05	–0.01	–3.14
G	M (dBc)	–97.6	–108.6	–125.3	–115.1	–124.6	–120.4	–126.4	–126.3	–126.2	–129.0
computed	Δφ (rad)	1.22	1.74	–1.51	–1.73	–0.00	1.91	0.01	–1.75	0.12	1.84
Y_N	M (dBc)	–104.9	–108.3	–126.3	–114.5	–125.9	–119.8	–126.4	–125.1	–128.2	–128.8
+correction	Δφ (rad)	0.79	1.66	2.46	–1.72	–1.45	1.67	2.13	–1.57	–1.63	1.64

Table 2. Result uncertainties

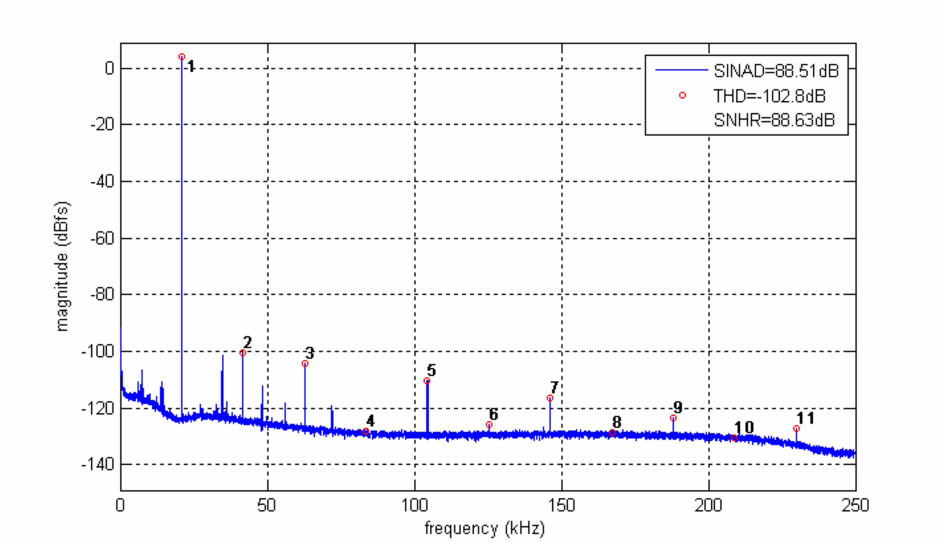
	harmonic component									
parameter	2nd	3rd	4th	5th	6th	7th	8th	9th	10th	11th
$\|G\|-2\|u_G\|$ (dBc)	–97.5	–108.4	–124.4	–114.7	–123.3	–119.3	–124.5	–124.5	–124.8	–126.9
$\|G\|+2\|u_G\|$ (dBc)	–97.7	–108.8	–126.2	–115.5	–126.2	–121.5	–129.0	–128.6	–128.0	–131.7
$2\|\mathrm{angle}(u_G)\|$ (rad)	1.98	0.06	0.54	0.05	0.48	0.50	0.23	0.34	0.13	0.10



a) signal $Y_F(j\omega)$: harmonic generator $G(j\omega)$ – filter $F(j\omega)$ – ADC $D(j\omega)$



b) input test sine-wave computed by the correction method



c) signal $Y_N(j\omega)$ with reconstructed fundamental: harmonic generator $G(j\omega)$ – filter $N(j\omega)$ – ADC $D(j\omega)$

Figure 2. Amplitude frequency spectra (Welch average of 63 spectra)