13th Workshop on ADC Modelling and Testing
Sep. 22-24, 2008, Florence, Italy

Noise shaping structures of band pass SD ADC and their impact on metrological parameters.

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Abstract- The bandpass sigma delta analog-digital converters (BP ΣΔ ADC) represent the unconventional ADCs, which are mostly embedded into digital communication systems or direct conversion of the voltage vector into the digital complex number. The structure of the noise shaping feedback impacts dominantly S/N ratio. The various feedback structures of the BP ΣΔ ADC designed from their low-pass prototype will be presented in the paper. The impact of the proposed structures on the basic metrological parameters of the BP ΣΔ ADC will be analyzed here.

I. Introduction

The main advantage of a ΣΔ ADC is flexibility of achieved resolution and noise shaping feedback structure. The simple circuit implementation with high degree of reprogramability of its binary blocks is another advantage of this structure. The basic structure of the ΣΔ modulator is transformable in the frequency domain. The most popular modification of basic low pass ΣΔ ADC is the band pass ΣΔ ADC.

The BP ΣΔ ADCs are mostly embedded into digital communication systems (DCS) in various applications. Their main task is to provide a frequency down conversion along with the conversion of chosen parameter of analog signal into digit. These converters are analog front end for DCS like software radios, UMTS, GSM and GPS systems. Another application of the BP ΣΔ ADC is targeted on the direct conversion of the complex harmonic signal at the input into the digital representation of its real and imaginary part. Such phase sensitive detection and analog-to -digital conversion is useful for processing signals from sensors where complex output impedance is modulated be two measured parameters mutually. High suppression of the quantisation noise around the central frequency is another advantage of implementation BP ΣΔ ADC.

The noise suppression of LP ΣΔ ADC depends on the noise shaping function. The increasing order of noise shaping structure is limited by stability constrains. The multiple noise shaping structures known as MASH ΣΔ ADC is the way how to avoid problems with stability. While there are many papers proposing various modifications of the noise shaping structure with ensured stability, the BP ΣΔ ADC are mostly based on the first order structure. Authors present the approach how to design various types of the noise shaping structures in the case of BP ΣΔ ADC. Moreover the improvement in quantization noise suppression is derived and experimentally evaluated.

The transfer functions of basic ΣΔ feedback structures of L-th order is represented by the formula (1). Corresponding circuit realisations are on [3].

$$Y(z) = X(z).z^{-1} + Q(z)\left(1 - z^{-1}\right)^{L} \quad (1) \qquad (1)$$

The transfer function of the MASH ADC structure is characterized by the formula (2). The noise components Q1,Q2 corresponds to the first and second comparator in the parallel structure. For transfer functions $F1=z^{-1}$ and $F2=(1-z^{-1})$ the quantization noise impact is eliminated.

$$\begin{aligned} Y(z) &= z^{-1}.F_1(z).X(z) + \left[F_1(z).\left(1 - z^{-1}\right) - F_1(z).z^{-1}\right]Q_1(z) + F_2(z).Q_2(z).\left(1 - z^{-1}\right) \\ &= \left| \begin{matrix} F_1(z) = z^{-1} \\ F_2(z) = 1 - z^{-1} \end{matrix} \right| = X(z)z^{-1} + Q_2(z).\left(1 - z^{-1}\right)^2 \end{aligned} \qquad (2)$$

Any other type of the ΣΔ modulator is achieved by the frequency transformation of the basic low pass transfer characteristic [3]. The frequency transformation from LP to BP is being performed by replacing of the term z-1 in the LP function by the (-z-2). The obtained BP structure has doubled order and final transfer functions are expressed by following equations.

$$Y(z) = -X(z).z^{-2} + Q(z)\left(1+z^{-2}\right)^{L} \qquad (3)$$

$$Y_{MASH}(z) = -z^{-2}F_1(z).X(z) + \left[F_1(z).\left(1+z^{-2}\right) + F_1(z).z^{-2}\right].Q_1(z) + F_2(z).Q_2(z).\left(1+z^{-2}\right) \qquad (4)$$

Corresponding circuits are shown on Fig.1. The integrator in the LP structure is being replaced by the resonator.

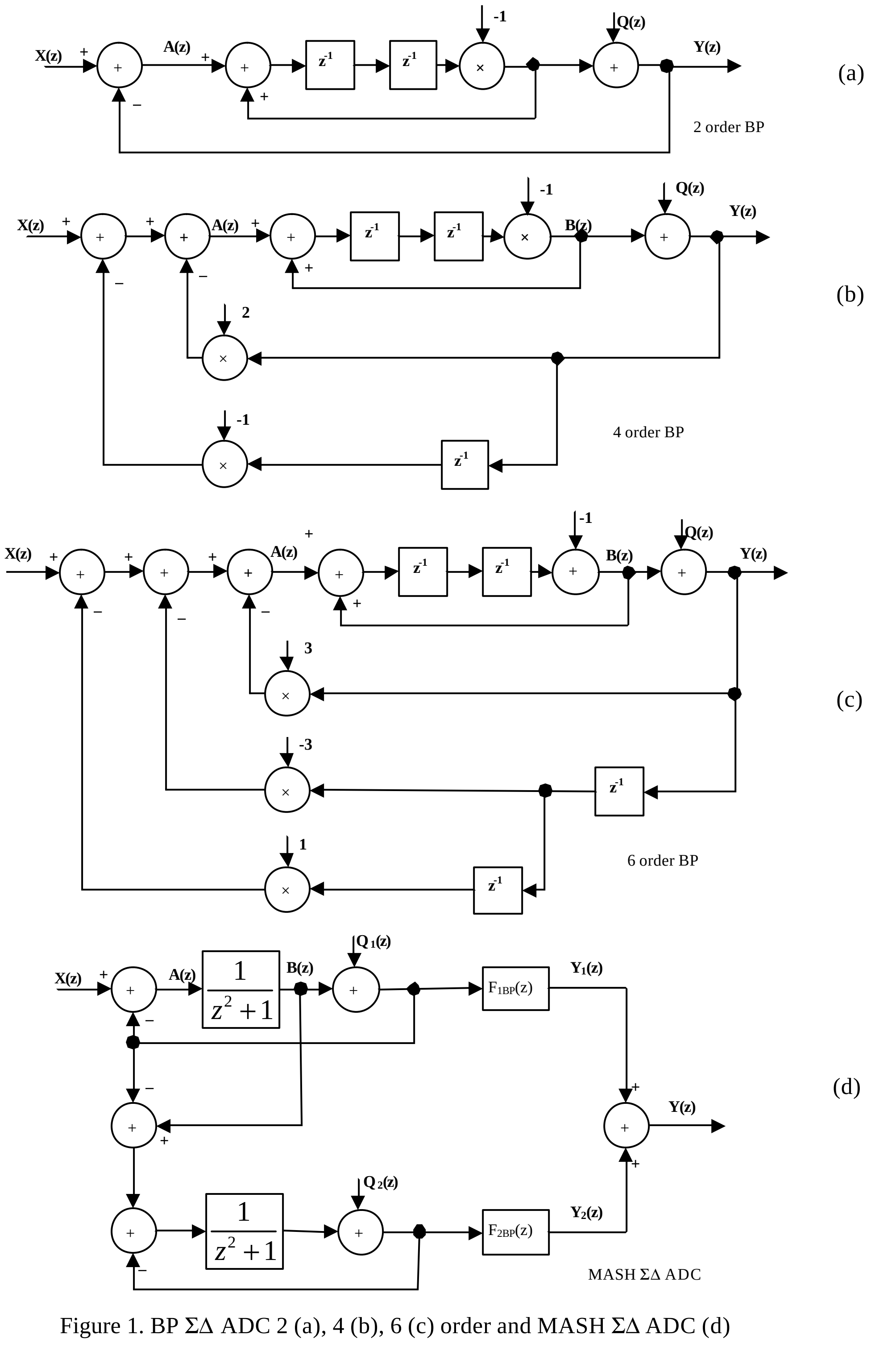

Figure 1. BP ΣΔ ADC 2 (a), 4 (b), 6 (c) order and MASH ΣΔ ADC (d)

II. The quantization noise reduction

The effect gain of the noise suppression is being determined by the resolution gain *R* caused by the shaping feedback and low pass filtering. The total resolution is expressed by the formula:

$$R = \log_2 \frac{u_{q,NS}}{u_q} = \log_2 \frac{u_{q,IN}}{u_q} + \log_2 \frac{u_{q,NS}}{u_{q,IN}} = R1 + R2 \qquad (5)$$

Where $u_{q,IN}$ is effective quantization noise after noise filtering, $u_{q,NS}$ is effective noise at the LP filter and u_q the same noise before filtering.

Let consider input signal with frequency band $\Delta w = \frac{w_S}{2}$ around central frequency $w_C = \frac{w_S}{4}$.

Oversampling ratio *OSR* $\Delta w_{out} = \frac{w_S}{2.OSR}$ determines signal frequency range at the output of the BP ΣΔ ADC. This frequency range determines the maximal modulating frequency carrying information.

Taking in the consideration the transfer functions (3) and (4) the spectral noise power density is determined by the formula:

$$S(w) = S_e(w).2.[1 + \cos 2wT_S]^{L\prime} = S_e(w).4.\cos^{2L} wT_S \qquad (6)$$

Where S_e is uniform noise power density of the quantizer. $e_1^2 = \frac{Q}{6.\Delta w_s} = \frac{u_q^2}{w_S/2}$

The noise power $u_{q,NS}$ after filtering by rectangular transfer function (Ideal Filter) in the frequency range $w_C \pm \frac{\Delta w}{2}$ is

$$u_{q,NS}^2 = e_1^2.\frac{1}{T_S}\int_{w_c-\Delta w}^{w_c+\Delta w} 4^L.\cos^{2L} wT_S\, dwT_S \qquad (7)$$

The resolution gain R1 after noise shaping to the uniform quantization noise from the same frequency range could be calculated from the spectrum in the frequency domain.

$$R1 = -\frac{1}{2}\log_2 \frac{1}{\Delta w}\int_{w_c-w_S/2}^{w_c+w_S/2} 4^L.\cos^{2.L} w.T_S\, dw \qquad (8)$$

Values of R1were calculated numerically (Table.1).

R1	OSR=4	OSR=16	0SR=64
L=1	0,3541	0,8607	1,4335
L=2	0,6060	1,6063	2,7114
L=3	0,8177	2,3095	3,9469

Table 1. Resolution gain R_1

The analytical value *R* was compared using the simulation of BP

The noise suppression after averaging FIR filter (Av FIR) with length of *OSR* samples is represented

$$u_{q,NS}^2 = \frac{u_q^2}{f_S/2}.\frac{4^L}{T_S}\int_{w_c-w_S/2}^{w_c+w_S/2} \frac{wT_S}{2}\sin^2\left(\frac{wT_S}{2}.OSR\right) dwT_S \qquad (9)$$

The expression is being calculated numerically. Effect of different filters on the resolution gain R caused by the noise shaping R1 and LP filtering R2 from the output of the BP ΣΔ modulator noise for parameter L=2 is shown in the Figure 2. Various types of the LP filters e.g. FIR filter, Bessel, Chebyshev and Butterworth filters of different orders have been studied.

As shown on the figure the averaging comb filter is not as good as the others considered filters. From all considered filters, the Butterworth filter provides the best results.

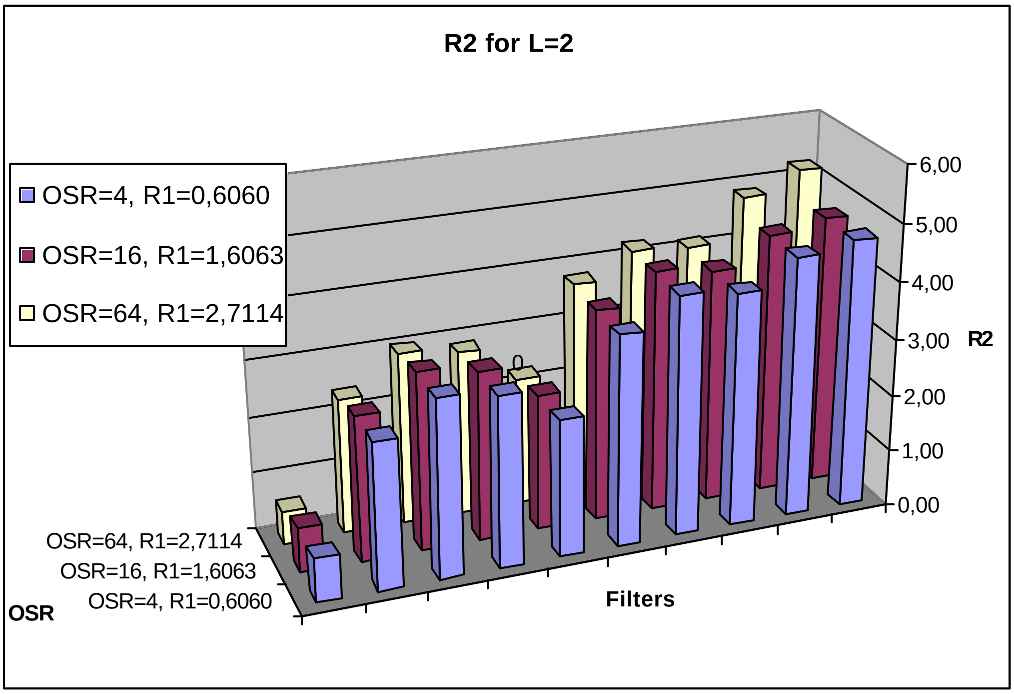

Figure 2. Resolution *R* for diferrent filters

In the next graph (Fig. 3) is shown the effect of parameter M on R1 and R2 for 2. order Butterworth filter.

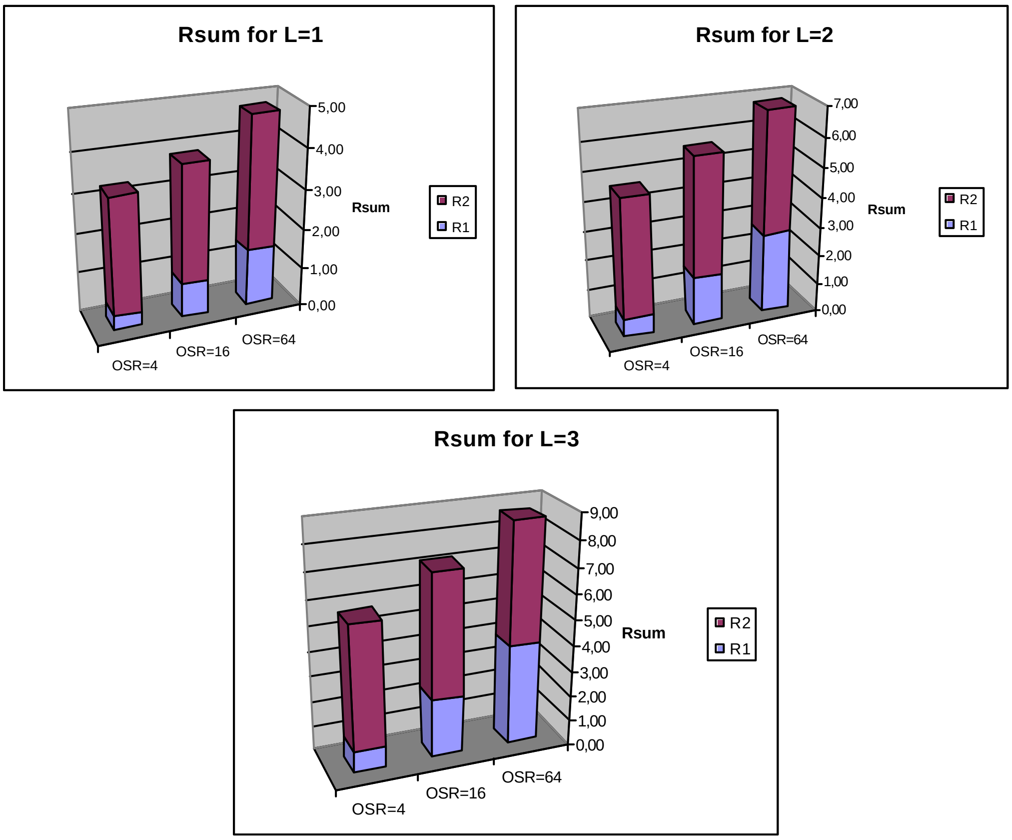

Figure 3. R1 and R2 for 2. order Butterworth filter

III. Experimental results

The mathematical analysis from the previous paragraph was performed under some simplification. First one is the consideration that quantization noise is ideally white. Second consideration is ideal transfer function of the LP filters at the modulator output. Third one is the inherent noise of electronic components in the converter. Besides thermal and flicker noise the switching blocks contribute to the final noise dominantly.

The noise figures improvement of the proposed BP final ΣΔ ADC has been studied using software simulations. The error model of the converter was developed in the LabVIEW environment. The noise suppression was calculated from the noise spectra for various orders of noise shaping feedback (Fig.4). The noise spectra at the output of the ΣΔ modulator and ΣΔ ADC are shown on Fig.4 a-c.

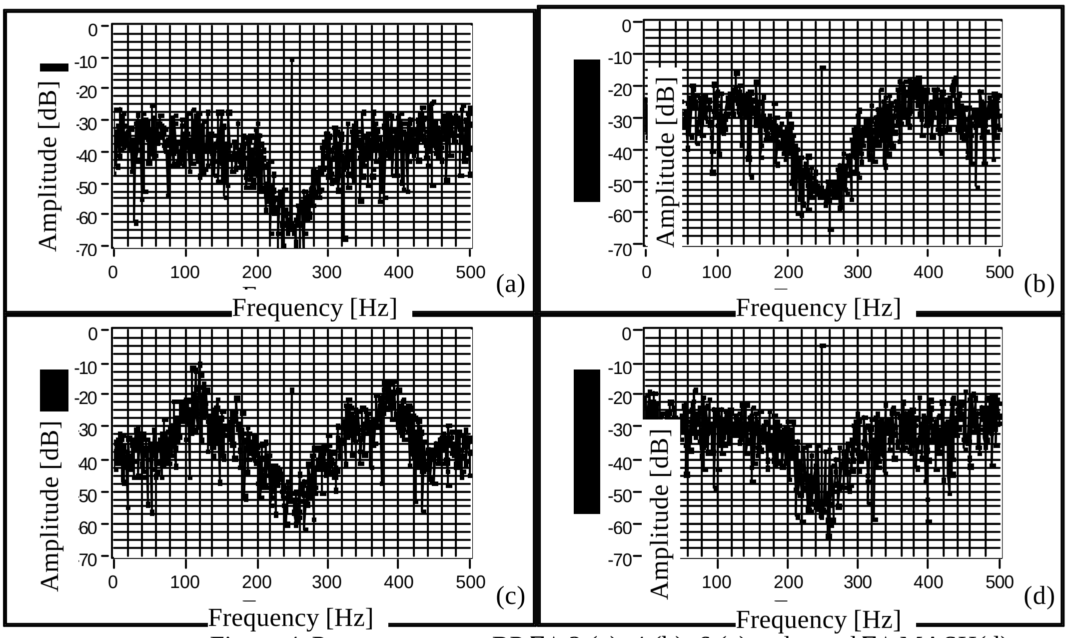

Figure 4. Power spectrum BP ΣΔ 2 (a), 4 (b), 6 (c) order and ΣΔ MASH(d)

Authors has designed real BP ΣΔ ADC on the programmable IC Cypress SoC . Its transfer function is shown on Fig.5. The difference to the ideal shape is caused by the additive switching noise. The final experimentally verified spectra shows satisfying conformity with the simulation results.

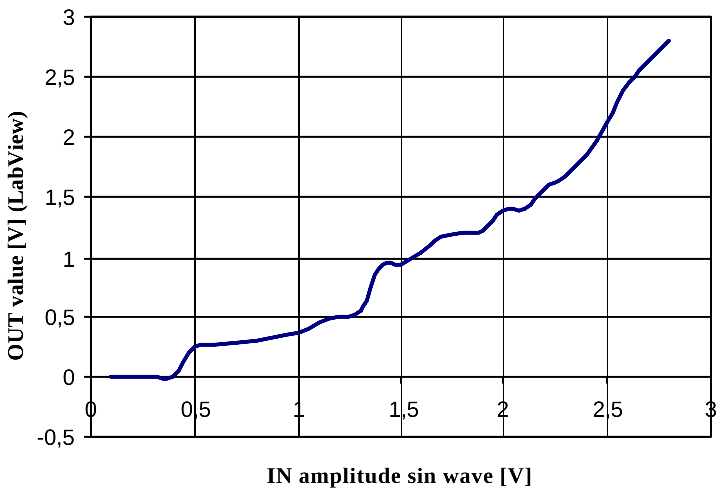

Figure 5. BP ΣΔ ADC transfer function

IV. Conclusions

As shown from the formula (1) and (3) ideal noise shaping function around zero frequency in LP ΣΔ ADC is the same as for the BP ΣΔ ADC around the central frequency. For real converters differences in the parameter representing noise suppression are caused by the switching noise mainly.

Acknowledgement
The work is a part of project supported by the Science Grant Agency of Slovak republic (No. 1/0103/08).

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