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New approximation models for processing exponential stimulus histogram test of ADC

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Abstract- The paper presents a new approach to processing of the cumulative code histograms obtained at ADC histogram testing by exponential stimulus signal. The authors proposed two new approximation models of cumulative and code histograms derived from the base model: recurrent model with rectangular rule and asymptotic model with the goal to simplify approximation process. The recurrent model is convenient for the implementation in digital signal processor because it uses only basic arithmetic operations. Estimation process using this model is also faster. The asymptotic model enables the fastest approximation. The models were experimentally compared by simulation using Levenberg–Marquardt and Newton iteration algorithms.

I. Introduction

Dynamic testing methods are often used for determination of ADC transfer characteristics. This estimated transfer characteristic describes how the applied input signal is digitized. ADC testing by the exponential shaped voltage was introduced in some papers ([1], [2], etc). The main advantage of such a signal is its easy circuit implementation. It is possible to use common capacitors on the market to test ADC with resolution at least up to 14 bits.

To find out ADC transfer characteristic the parameters of stimulus signal have to be determinate by estimation. The authors have chosen the estimation from the cumulative histogram, because the estimation from the time record requires calculation of two stimulus parameters – time constant and constant *B* (1). More over the approximation process require a big number of points – usually all samples in the record. On the other hand the estimation from cumulative histogram requires relatively small number of points equals to 2^N-2 and the only result is constant *B*. Moreover the cumulative histogram is more resistant to the superimposed noise and harmonic interferences.

In praxis the parameter can be estimated only numerically. The numerical estimation is a complex task and requires sufficient computing power what is in contrary to manifested simplicity of exponential stimulus test method. Therefore the authors suggested some new approaches derived from cumulative histogram analytical model.

Let us assumed that $y(t)$ is the exponential stimulus signal (Fig. 1.) of ADC and it can be described by equation as follows:

$$y(t) = (F_2 - B)\cdot e^{-\frac{t}{\tau}} + B, \tag{1}$$

where τ is the time constant of exponential pulse, F_1 and F_2 determines full-scale input range of ADC under test and B is limit value of exponential signal for $t \to \infty$.

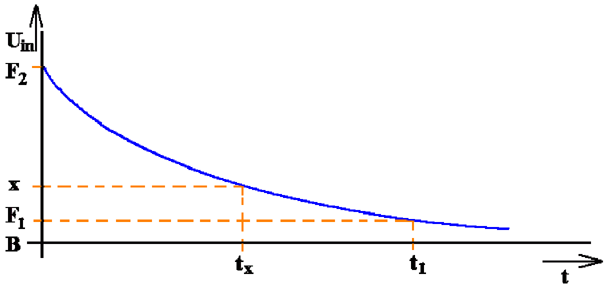

Fig. 1. Exponential stimulus signal described by equation (1)

II. Analytical model of cumulative and code histogram

Based on exponential stimulus time model (1), distribution function *P*(*x*) specifying the cumulative histogram shape of stimulus can be simply derived according to Fig. 1.:

$$P(x) = P(X < x) = P(X \in \langle F_1, x \rangle) = P(t \in \langle t_x, t_1 \rangle) = \frac{t_1 - t_x}{t_1}, \qquad x \in \langle F_1, F_2 \rangle, \tag{2}$$

where

$$t_x = -\tau \cdot \ln\left(\frac{x - B}{F_2 - B}\right), \quad t_1 = -\tau \cdot \ln\left(\frac{F_1 - B}{F_2 - B}\right). \tag{3}$$

Hence

$$P(x) = \frac{t_1 - t_x}{t_1} = C \cdot \ln\left(\frac{x - B}{F_1 - B}\right), \tag{4}$$

where

$$C = \frac{1}{\ln\left(\frac{F_2 - B}{F_1 - B}\right)}. \tag{5}$$

The probability density function is defined as follows:

$$p(x) = P'(x) = C \cdot \frac{1}{x - B}. \tag{6}$$

The expected cumulative and code histograms results from the distribution function and probability density function. By applying substitutions $L = \frac{F_1 - B}{Q}$ and $x_k = k \cdot Q + F_1$ we can gain expressions (7) and (8). Constant *Q* is code bin nominal width ($Q = \frac{F_2 - F_1}{2^N - 1}$) and x_k is *k*-th quantization level.

Hence cumulative and code histogram can be derived:

$$H_c(k) = C \cdot \ln\left(\frac{x_k - B}{F_1 - B}\right) = C \cdot \ln\left(\frac{F_1 - B + k \cdot Q}{F_1 - B}\right) = C \cdot \ln\left(\frac{L + k}{L}\right), \tag{7}$$

and

$$H(k) = (H_c(k))' = C \cdot \frac{1}{L + k}, \qquad k = 1, 2, \ldots, 2^N\text{-}2. \tag{8}$$

This model is exact mathematical representation of ideal histogram. However, there are other approximation models hereinafter that can save computing time in process of estimation parameter *B* and that have comparable accuracy with hereinbefore described exact model.

III. Recurrent model of cumulative and code histogram using rectangular rule

The probability of the *k*-th transient level can be derived as follows:

$$H(k)=\int_{x_{k-1}}^{x_k} p(x)dx = C\cdot\ln\left(\frac{x_k-B}{F_1-B}\right)-C\cdot\ln\left(\frac{x_{k-1}-B}{F_1-B}\right)=C\cdot\ln\left(\frac{x_k-B}{x_{k-1}-B}\right), \qquad (9)$$

where

$$k = 1, 2, \ldots, 2^N\text{-}2, x_0 = x_1\text{-}Q. \qquad (10)$$

The equation (9) requires to calculate the integral (the area under the function p(x)) using function ln(.). To simplify this calculation the area may be approximated by the area of rectangular with height equal to $p\left(\frac{x_k+x_{k-1}}{2}\right)$ (Fig. 2). This substitution is called rectangular rule hereinafter.

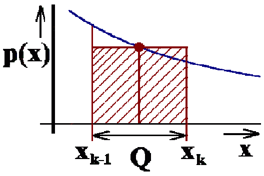

Fig. 2. Rectangular rule applied on the probability of the *k*-th transition level.

Using the rectangular rule the approximation of *H*(*k*) is as follows:

$$\hat{H}(k)=p\left(\frac{x_k+x_{k-1}}{2}\right)\cdot\left(x_k-x_{k-1}\right)=Q\cdot p\left(\frac{x_k+x_{k-1}}{2}\right)=\frac{Q\cdot C}{\frac{x_k+x_{k-1}}{2}-B}, \qquad (11)$$

Hence

$$\hat{H}(k)=\frac{\hat{D}}{L+k-\frac{1}{2}}, \qquad (12)$$

where

$$\hat{D}=\frac{1}{\sum_{k=1}^{2^N-2}\frac{1}{L+k-\frac{1}{2}}}. \qquad (13)$$

Consequently the approximation of cumulative histogram is as follows:

$$\hat{H}_c(k)=\hat{H}_c(k-1)+\hat{H}(k), \qquad \hat{H}_c(0)=0, \qquad k = 1, 2, \ldots, 2^N\text{-}2. \qquad (14)$$

The main advantage of this model is that formula (14) is a recurrent function using only the basic mathematical operations. Using recurrent process (14) approximation of the cumulative histogram can be calculated faster than using basic formula (7).

IV. Asymptotic model of cumulative and code histogram

Another computing acceleration can be achieved by an approximation ([3]) of the sum in (13).
Let us suppose that *L* is an integer value. Then

$$\begin{aligned}\frac{1}{\hat{D}}&=\sum_{i=1}^{2^N-2}\frac{1}{L+i}=\frac{1}{L+1}+\ldots+\frac{1}{L+2^N-2}=\\&=\left(1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{L}\right)+\frac{1}{L+1}+\ldots+\frac{1}{L+2^N-2}-\left(1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{L}\right)=S_{L+2^N-2}-S_L,\end{aligned} \qquad (15)$$

where

$$S_n=1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{n}=\sum_{i=1}^{n}\frac{1}{i}=\hat{S}_n+\frac{\varepsilon_n}{120\cdot n^4}, \qquad 0<\varepsilon_n<1, \qquad (16)$$

and

$$\hat{S}_n = \ln(n) + \gamma + \frac{1}{2\cdot n} - \frac{1}{12\cdot n^2}\,, \tag{17}$$

and γ is the Euler's constant which is expressed as follows:

$$\gamma = \lim_{n\to\infty}\left(\left(\sum_{i=1}^{n}\frac{1}{i}\right) - \ln(n)\right) \cong 0{,}5772156649... \tag{18}$$

Finally

$$\frac{1}{\tilde{D}} \cong \hat{S}_{L+2^N-2\frac{1}{2}} - \hat{S}_{L-\frac{1}{2}} = \ln\left(\frac{L+2^N-2\frac{1}{2}}{L-\frac{1}{2}}\right) + \frac{1}{2}\cdot\left[\frac{1}{L+2^N-2\frac{1}{2}} - \frac{1}{L-\frac{1}{2}}\right] - \frac{1}{12}\cdot\left[\frac{1}{\left(L+2^N-2\frac{1}{2}\right)^2} - \frac{1}{\left(L-\frac{1}{2}\right)^2}\right]. \tag{19}$$

Note, that formula (19) can be used also for real *L* ([3]).

The error of the asymptotic model ($\left|S_n - \hat{S}_n\right|$) is less than $\frac{1}{120\cdot n^4}$ (see equation (16)).

V. Experimental comparison of the models

The first approximations utilized Levenberg–Marquardt and Newton iteration algorithms of least–square fit method. All simulations were performed in LabWindows/CVI software by National Instruments. Levenberg–Marquardt iteration algorithm is included in library of this software development environment. Disadvantage of included algorithm is its impertinence for recurrent function implementation as required in expression (14). Non-recurrent functions were used in this case:

$$\hat{H}_c(k) = \hat{D}\cdot\sum_{k=1}^{k}\frac{1}{L+k-\frac{1}{2}}\,, \qquad k = 1, 2, \ldots, 2^N\text{-}2, \tag{20}$$

and

$$\tilde{H}_c(k) = \ln\left(\frac{L+k-\frac{1}{2}}{L-\frac{1}{2}}\right) + \frac{1}{2}\cdot\left[\frac{1}{L+k-\frac{1}{2}} - \frac{1}{L-\frac{1}{2}}\right] - \frac{1}{12}\cdot\left[\frac{1}{\left(L+k-\frac{1}{2}\right)^2} - \frac{1}{\left(L-\frac{1}{2}\right)^2}\right]. \tag{21}$$

The other iteration algorithm (Newton) was programmed as a subroutine. The recurrent function (14) was used here. Some parameters of estimation process are shown in Table 1. and Table 2..

8-bit ADC, B_{real}=-100, starting point B=-10, iteration condition $|B_{n+1}-B_n|$>1E-9

	Time of approx. process	Standard deviation of cumulative histogram approx.	Relative error of estimated parameter B	Number of iterations
Method 1	2.41 ms	2.6996E-05	1.2456E-05	6
Method 2	84.6 ms	2.7257E-05	2.2242E-05	6
Method 3	7.5 ms	2.7257E-05	2.2242E-05	6
Method 4	1.1 ms	2.6996E-05	1.2456E-05	11
Method 5	0.5 ms	2.7257E-05	2.2242E-05	11
Method 6	0.35 ms	2.7257E-05	2.2243E-05	11

Table. 1. Approximation results of suggested methods on 8-bits ideal ADC

14-bit ADC, B_{real}=-1E4, starting point B=-10, iteration condition $|B_{n+1}-B_n|$>1E-6

	Time of approx. process	Standard deviation of cumulative histogram approx.	Relative error of estimated parameter B	Number of iterations
Method 1	284 ms	4.5329E-05	3.4389E-06	9
Method 2	440 s	4.5329E-05	3.4399E-06	9
Method 3	624 ms	4.5329E-05	3.4399E-06	9
Method 4	98 ms	4.5329E-05	3.4390E-06	17
Method 5	49 ms	4.5329E-05	3.4399E-06	20
Method 6	24 ms	4.5329E-05	3.4399E-06	16

Table. 2. Approximation results of suggested methods on 14-bits ideal ADC

Where,
Method 1 is Levenberg–Marquardt iteration algorithm using model (7), (5).
Method 2 is Levenberg–Marquardt iteration algorithm using model (20), (13),
Method 3 is Levenberg–Marquardt iteration algorithm using model (21), (19),
Method 4 is Newton iteration algorithm using model (7), (5),
Method 5 is Newton iteration algorithm using model (14), (12), (13),
Method 6 is Newton iteration algorithm using model (14), (12), (19).

VI. Conclusions

The paper presents some new approximating models of cumulative and code histograms for exponential stimulus signal: recurrent model using rectangular rule and asymptotic model. The main advantage of these models is acceleration of estimation process of parameter B. These models were tested and compared with the basic exact model by simulations. In simulations two iteration algorithms – Levenberg–Marquardt and Newton were used. The achieved results show that the fastest model is the asymptotic model (eq. (14), (12), (19)). In contrary to the expectation the Newton iteration algorithm was faster than Levenberg–Marquardt algorithm. Newton algorithm uses many more iterations to achieve the similar error. The recurrent model (eq. (14), (12), (13)) is convenient for the implementation in DSP (digital signal processor) because it uses only basic arithmetic operations. This model is also faster than analytical one (eq. (7), (8), (5)) in estimation process at low approximation error.

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