Noise Influence On Exponential Histogram ADC Test

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Abstract- This paper deals with some error effects caused by additive noise at analog-to-digital converters (ADCs) testing based on the histogram method and the exponential shape of input testing signal. The histogram method with exponential signals has been an alternative test method for ADC developed by the author. Here, the theoretical analysis of some errors in estimation of code bin width and quantisation levels caused by additive input Gaussian noise is performed. The theoretical results are verified by simulations. The acquired results are compared with the analogues ones for sinewave and Gaussian noise input test signals.

I. Introduction

Analog-to digital converter (ADC) testing is expensive and time consuming process which needs high quality instrumentation according the standard testing methods ([1], [2], [3]) This fact leads to suggestion of the new non standardised methods based on different type of calibrating signal such as Gaussian noise ([4], [5]) or exponential pulses [6]. The main advantage of the last mentioned method based on exponential stimulus pulses and consecutive processing of the acquired ADC code histogram is the simple realisation of the stimulus generating circuit [6].

The presence of the additive input noise can deteriorate the results of ADC testing using any test methods. Analyses of noise influence were made for sinewave in [7] and [8] and for Gaussian noise stimulus in [9]. This paper is focused on determination of errors in estimations of code bin width and transition levels resulting from additive input noise influence at exponential stimulus histogram test method. The exponential stimulus signal (Fig. 1.) can be described by:

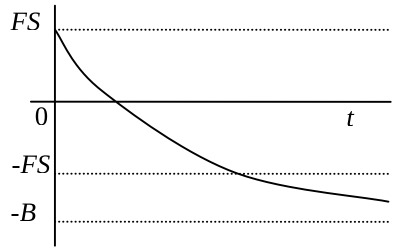


Figure 1. Exponential stimulus signal.

$$x(t) = (FS + B)\exp\left(-\frac{t}{\tau}\right) - B \ , \qquad (1)$$

where τ is the time constant of exponential pulse, *FS* and –*FS* determine full-scale input range of ADC under test and -*B* is limit value of exponential signal for $t \to \infty$. The distribution function *P*(*x*) and the density *p*(*x*)for such signal according the [6] are:

$$P(x) = \begin{cases} 0 \\ C \ln \dfrac{B + FS}{B + x} + 1, \\ 1 \end{cases} \quad p(x) = \begin{cases} 0 & \text{for} & x < -FS \\ \dfrac{-C}{B + x} & \text{for} & -FS \le x \le FS \\ 0 & \text{for} & x > FS \end{cases} \qquad (2)$$

where *C* is constant, $C = -1 \Big/ \ln \dfrac{B + FS}{B - FS}$

II. Influence of additive noise

Let the exponential signal is deteriorate by the additive Gaussian noise with bias μ=0, variance σ^2, and density function *g*(*x*) Such a noise can cause errors in estimation $\hat{T}_k$ of ADC code transition levels T_k

and estimation $\hat{W}_k$ of code bin width W_k. (k=0, 1, 2^N-1, N is nominal number of ADC bits) and they are assessed thereinafter.

A. Error in estimation of code bin width

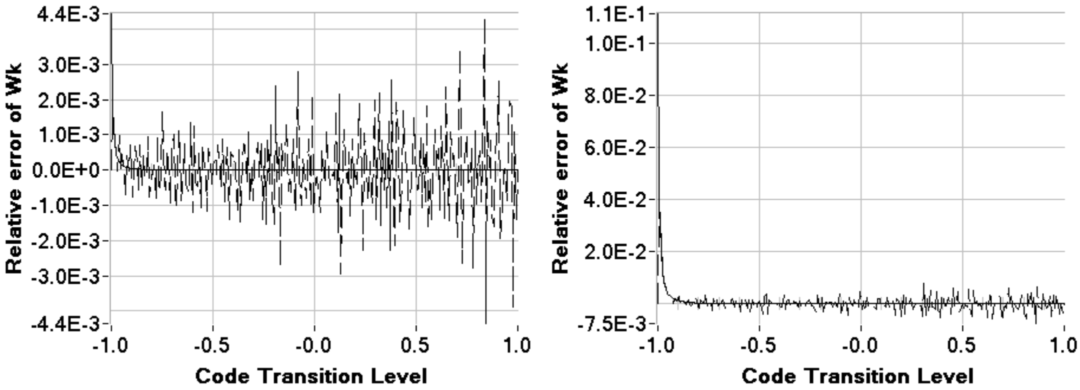
The density function of sum of the calibrating signal (1) and the noise is given by the convolution $f(x)=p(x)*g(x)$ that can be approximated according to the lemma 1 in [7] and substituting $p(x)$ from (2):

$$f(x) = p(x) * g(x) \cong p(x) + \frac{\sigma^2}{2} p''(x) = -C \frac{(B+x)^2 + \sigma^2}{(B+x)^3} \tag{3}$$

The relative error in estimation of code bin width is given

$$E_{\hat{W}_k} = \frac{\hat{W}_k - W_k}{W_k} = \frac{\hat{W}_k}{W_k} - 1 \cong \frac{f(x)}{p(x)} - 1 \cong \frac{\sigma^2}{(B+x)^2} \le \frac{\sigma^2}{(B-FS)^2} \tag{4}$$

Maximal error is reached for x closing to the $-FS$ and can be minimised by increasing B. The analytically predicted error was verified by simulation performed in LabVIEW and LabWindows/CVI by National Instruments. It proved a good match between the predicted and the simulation results as it can be seen for some examples in Fig. 2.



a. b.

Figure 2. Comparison of relative error of W_k estimation acquired from the analytical prediction (4) and from simulation, number of samples 100000, number of test repetitions 500. The figure a. shows result for σ=0,1 and the figure b. for σ=0,5.

Let the maximal acceptable relative error of testing due to the additive noise for estimation of code bin width be β, e.g. β=0.1. Then the stimulus generating circuit must generates the signal with constant B as follows

$$\beta \ge \frac{\sigma^2}{(B-FS)^2} \quad \Rightarrow \quad B \ge \frac{\sigma}{\sqrt{\alpha}} + FS \cong 3{,}16\,\sigma + FS \tag{5}$$

The result shows that the need to overdrive of the ADC input to ensure acceptable precision of testing spoiled due to the additive noise is relative small and it is only a little larger than overdrive needed for sinewave which is according to [7] for the same precision equal to 1,94σ. If the second derivation of density function is not constant and it is rapidly changed what occurs for relatively small B, the real error will be larger than predicted by (4).

B. Error in estimation of code transition levels

The estimation $\hat{T}_k$ of ADC code transition levels T_k can be done from the cumulative histogram built form M samples which is proportional to the distribution function.

$$\hat{T}_k \cong \hat{x} = (B+FS)\exp\left(-\frac{H_c(k)}{MC} - \frac{1}{C}\right) - B = (B+FS)\exp\left(-\frac{F(x)}{C}\right) - B \tag{6}$$

The distribution function of sum of the exponential stimulus signal and the noise $F(x)$ is given by the convolution of distribution function of signal $P(x)$ and density function of noise $g(x)$, i.e. $F(x)=P(x)*g(x)$. According to [7], the convolution can be approximated as follows:

$$F(x) = P(x) + \frac{\sigma^2}{2} P''(x) \tag{7}$$

Substituting (7) in (6) gives

$$\hat{T}_k \cong \hat{x} \cong (B + FS)\exp\left(-\frac{P(x) + \frac{\sigma^2}{2}P''(x)}{C}\right) - B \tag{8}$$

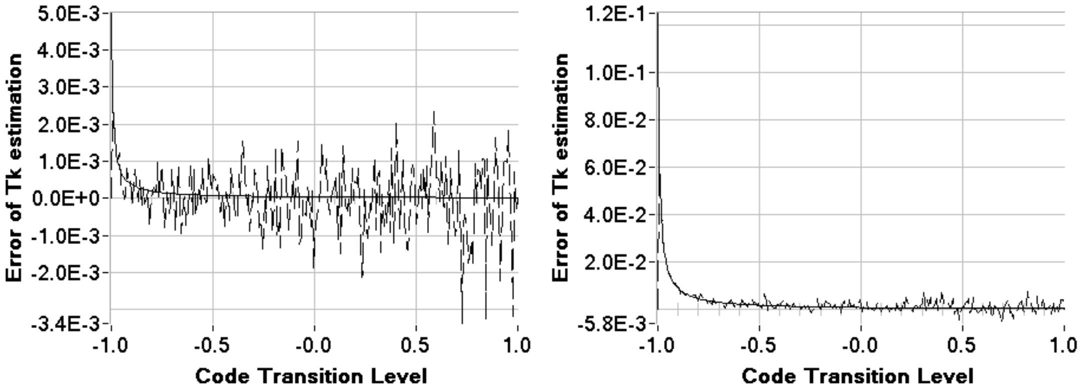
Taking a first-order Taylor expansion of exp(.) function about $-\frac{P(x)}{K}$ and supposing that $\frac{\sigma^2}{2}P''(x)$ is relatively small lead to

$$\hat{x} \cong (B + FS)\left(\exp\left(-\frac{P(x)}{C}\right) - \frac{\sigma^2}{2C}P''(x)\exp\left(-\frac{P(x)}{C}\right)\right) - B =$$
$$= (B + FS)\exp\left(-\frac{P(x)}{C}\right)\left(1 - \frac{\sigma^2}{2C}P''(x)\right) - B \tag{9}$$

Substituting (2) gives

$$\hat{x} \cong \frac{\sigma^2}{2}\frac{1}{B + x} + x \quad \Rightarrow \quad \hat{x} - x = \frac{\sigma^2}{2}\frac{1}{B + x} \tag{10}$$

The maximal error of estimation is reached for x closing to $-FS$ and, analogously to the error of code bin width estimation, the error can be minimised by increasing B. A good match between the predicted and the simulation results was found as it can be seen for some examples in Fig. 3.



a. b.

Figure 3. Comparison of errors in T_k estimation acquired from the analytical prediction (10) and from simulation number of samples 100000, number of test repetitions 500. The figure a. shows result for σ=0,1and the figure b. forσ=0,5.

Let the maximal acceptable error of testing due to the additive noise for estimation of code transition level be β, e.g. β=0.1LSB. Then the stimulus generating circuit must generate the signal with:

$$\beta = \hat{x} - x \geq \frac{\sigma^2}{2}\frac{1}{B + x} \quad \Rightarrow \quad B \geq \frac{\sigma^2}{2\alpha} + FS = 5\frac{\sigma^2}{LSB} + FS \tag{11}$$

The result shows that the need to overdrive of the ADC input to ensure acceptable precision of testing spoiled due to the additive noise is relative small. If the effective value of the noise is smaller or comparable to 1LSB, the overdrive of 5LSB seems to be enough to overcome the decreasing test precision caused by the noise. This overdrive is two times lager than the overdrive needed for sinewave which is according to [7] for the same precision and the noise equal to 2.5LSB. If the second derivation of density function is not constant and it is rapidly changed what occurs for relatively small B, the real error will be larger than predicted by (10).

C. Variance of transition code levels estimation

Let suppose the uniform sampling, i.e. the time interval T_{MEAS} while the noiseless exponential signal is within the ADC input range divided into M equal time steps with width Δt. The time precisely corresponding to the transition code level T_k is $(M-(n+\alpha))\Delta t$, where n is an integer and α is a number between 0 and 1. Then the probability that $H_c(k) = n+1$ is α and $H_c(k) = n$ is $1- \alpha$. The mean value of such distribution is $n+\alpha$ and variance $\alpha(1- \alpha)$. The density of samples around the transition level T_k in Volts is given by (2). Using these facts we will get the variance of the transition level estimation

$$\sigma^2_{\hat{T}_k} = \alpha(1-\alpha)\frac{1}{M^2 p^2(T_k)} = \alpha(1-\alpha)\frac{1}{M^2 C^2}(T_k + B)^2 \qquad \alpha = \left\langle \frac{T_{MEAS}}{M} t_s \right\rangle, \qquad (12)$$

where $\langle . \rangle$ is the fractional part operator and t_s is a sampling instance.

Now let a Gaussian noise be added to the input exponential testing signal. Let consider the probability that a sample is inside the time interval for a cumulative histogram bin at a distance x in Volts, from the edge of the interval. If it is from the right edge then $G(x)$ is the probability that the noise voltage is smaller or equal to x. If it is distance from the left edge, then $G(x)$ is the probability that the noise voltage is bigger or equal to x. In either case

$$G(x) = \frac{1}{\sqrt{2\pi\sigma}} \int_{-\infty}^{x} \exp\left(-\frac{v^2}{2\sigma^2}\right) dv \qquad (13)$$

Let c_j be random variable that takes the value one if j-th sample point is recorded in the cumulative histogram in question and takes the value zero if it is recorded outside. Then the variance $v(x)$ of random variable c_j is $G(x)$ $(1- G(x))$. Let λ be a density in Volts of samples around the border of sampling interval for code bin k. Using (2) we obtain:

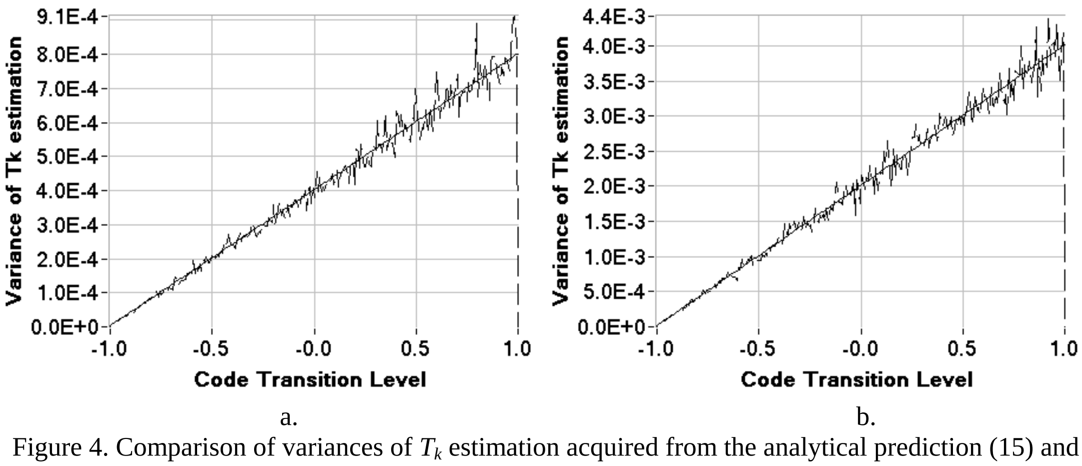
$$\lambda = \frac{1}{M\,|p(T_k)|} = \frac{(T_k + B)}{MC} \qquad (14)$$

Because of two regions around the border of code bin contributing to the variance the factor 2 must be added to the variance of transition level estimation. Finally combining the previous facts we obtain:

$$\sigma^2_{\hat{T}_k} = 2\lambda \int_0^{\infty} v(x)dx = \frac{2}{M\,.\,p(T_k)} \int_0^{\infty} v(x)dx = -\frac{1.13}{2}\frac{\sigma}{MC}(T_k + B) \qquad (15)$$

where according to [7] $\int_0^{\infty} v(x)dx = \int_0^{\infty} G(x)(1-G(x))dx = \frac{1.13}{4}\sigma$

The analytical prediction of variance (15) proved the good consonance with simulation results as it can be seen for some examples in Fig 4.



a. b.

Figure 4. Comparison of variances of T_k estimation acquired from the analytical prediction (15) and from simulation, number of samples 100000, number of test repetitions 500. The figure a. shows result for σ=0,1 and the figure b. for σ=0,5.

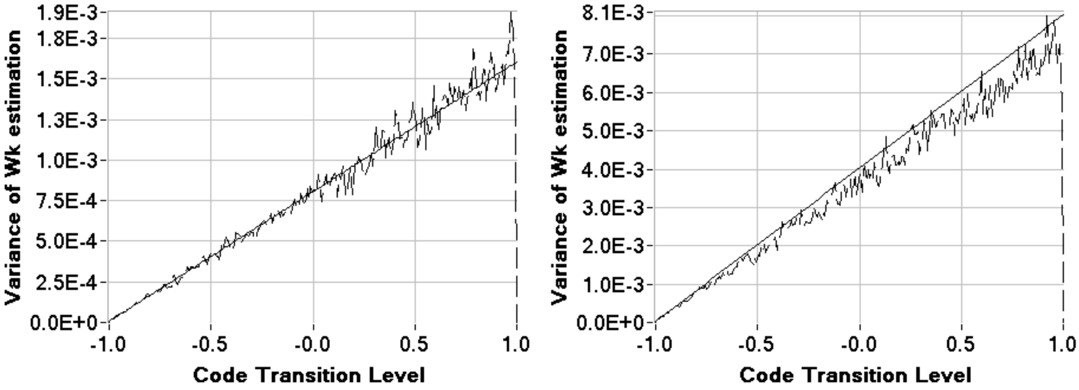
The total variance of transition level estimation is given by addition of equation (12) and (15) because of independence of these two effects. The dominant component of variance at a noise presence is given by (15) because of square of *M* in denominator in (12).

D. Variance of code bin width estimation

A code bin width is the difference between two adjacent code transition levels. If the code bin width is larger than twice the noise level then the errors in the code transition level can be supposed to be nearly independent and that's why the variance of code bin width can taken twice variance of code transition levels estimation, i.e.

$$\sigma^2_{\hat{W}_k} = -1.13\frac{\sigma}{MC}(T_k + B) \qquad (16)$$

The analytical prediction of variance (16) proved the good consonance with simulation results as it can be seen for some examples in Fig 5.



a. b.

Figure 5. Comparison of relative error of code bin width estimation acquired from the analytical prediction (4) and from simulation for, number of samples 100000, number of test 500. The figure a. shows result for σ=0,1 and the figure b. for σ=0,5.

If the noise level is higher we would have to take in account the covariance of adjacent levels but because of natural low past filtering in generation of slow exponential stimulus the case of large noise seems to be very rare in praxis and is out of our focus in this paper.

II. Comparison of noise influence on histogram test for sinewave and exponential signal

The variance of transition level estimation for sinewave histogram test according to [7] and [8] is:

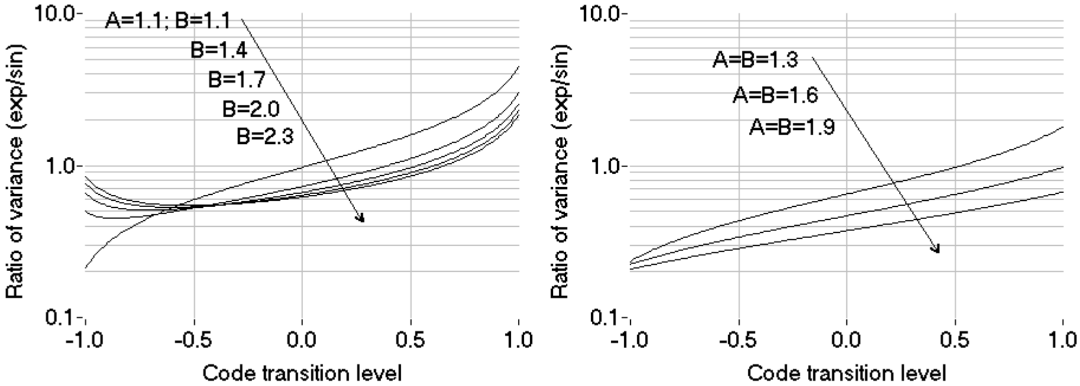
$$\sigma^2_{SHT} = \alpha(1-\alpha)\frac{\pi^2}{M^2}\left(A^2 - \hat{T}_k^2\right) + \frac{1.13\pi}{2}\frac{\sigma}{M}\sqrt{A^2 - \hat{T}_k^2} \qquad (17)$$

$$\alpha_k = \left\langle \frac{2\psi_k}{\Delta\varphi} \right\rangle, \quad \Delta\varphi = \frac{2\pi}{M}, \quad \psi_k = \arccos\left(-\frac{\hat{T}_k}{A}\right), \quad \hat{T}_k = T_k - d$$

The dominant component at the presence of noise in (17) is its second part [9]. The ratio of variance for sinewave histogram test σ^2_{SHT} and variance for exponential stimulus histogram test σ^2_{EHT} for DC component of sinewave *d*=0 is

$$\frac{\sigma^2_{EHT}}{\sigma^2_{SHT}} = \frac{-\frac{1.13}{2}\frac{\sigma}{MC}(T_k + B)}{\frac{1.13\pi}{2}\frac{\sigma}{M}\sqrt{A^2 - T_k^2}} = \frac{-(T_k + B)}{\pi C\sqrt{A^2 - T_k^2}} \qquad (18)$$

The equation (18) indicates that this ratio does not dependence on noise level and number of samples but it dependences only on shape parameters of test stimulus signals. The minimum can reached for $T_k = -FS$ or anywhere within the ADC input range as it can be seen from some examples in Fig. 6a. It means that neither sinwave nor exponential stimulus have principally better robustness in relation to additive noise.



a. b.

Figure 6. Comparison of variance ratio for exponential and siwave histogram test according to (18). Fig. 6a shows situation for A=1,1FS and various B, Fig. 6b. shows situation for various value A=B.

To simplify partially the comparison let $B=A$. Analysing (18) one can simple find that the minimum is always reached for $T_k=-FS$ and maximum for $T_k=FS$. If $B=A>1.581$ the ratio (18) is always smaller than 1 for any T_k, i.e the variation of code transition level estimation for the exponential stimulus test is always smaller than for sinewave test in such conditions. If $B=A<1.581$ the smaller variance is achieved for lower T_k by exponential stimulus test and by sinewave test for higher $T_{k.}$ (Fig. 6b).

III. Conclusions

The influence of additive noise on the precision of estimation of code bin width and transition code levels for the exponential stimulus histogram test method of ADC has been analysed. Analytical expressions for estimation errors have been derived and verified by simulation with good consonance. The results show that the errors are relatively small and closed to those for sinewave test. Neither sinewave nor exponential stimulus have principally and generally better robustness in relation to additive noise. The test errors due to additive noise can be minimised by convenient construction of stimulus signal generating circuit with large value of limit B of exponential signal.

Acknowledgement

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