Improved Residual Analysis in ADC Testing

István Kollár

Dept. of Measurement and Information Systems
Budapest University of Technology and Economics
H-1521 Budapest, Magyar tudósok krt. 2., HUNGARY
Tel.: +36 1 463-1774, fax: +36 1 463-4112
Email: kollar@mit.bme.hu

Abstract – Analog-to-digital converters are commonly tested by applying a pure sine wave at their inputs. Since the exact parameters of the sine wave are very difficult to precisely obtain, an indirect method is used: the measured samples are used to determine the sine wave which best fits them. The ADC is characterized then by analysis of the differences between the samples and the sine wave. This is described in the IEEE standards 1241-2000 and 1057-1994.

The generally applied method for the fit is least squares (LS). If the error (the deviation of the ADC output samples from the true sine wave) is a random sequence, with independent, identically distributed zero-mean samples, the LS fit effectively averages it out. However, when the quantization errors dominate in the observations, this is not true. Strange, systematic errors may arise, which cannot be averaged out. The paper examines these errors, and makes suggestions how to reduce them.

Keywords –IEEE Standard 1241-2000, IEEE Standard 1057-1994, ADC test, analog-to-digital converter, sine wave fitting, least squares, effective number of bits, ENOB.

I. Introduction

The theoretically correct testing method of input-output devices is to execute an experiment, measure the input and output signals, and to analyse the interrelation of them. This is the basic idea of the sine wave testing of ADC's: sine wave excitation is applied at the input of the ADC, and the measured samples are then analysed. However, accurate enough measurements of the input signal are often very difficult and expensive to execute. Therefore, the input, which can be accurately described by a few parameters, like a sine wave can be described by its amplitude, phase, frequency, and dc value[1], is not directly measured. The parameters are determined from the output sample series. How to do this, is described in the IEEE standards 1241-2000 and 1057-1994 [1-5]. When the frequency is not known, the so-called the 4-parameter method is applied, when the frequency is known (measured, or known from the setting of the sine generator), the 3-parameter method is performed. Then, analysis is continued by the investigation of the residual errors.

The idea of these methods is as follows. The data are not only quantized by the ideal ADC characteristic, but also prone to (usually small) ADC errors. We *fit them* with a sine wave, by adjusting its parameters. The fit is performed by setting the parameters to minimize a global measure of the level of deviations of the output data from the corresponding sine wave samples. In the standard, the sum of the squares of the errors is minimized, that is, a *least squares* (LS) fit is performed.

$$\min_{A,B,C,\omega} \sum_{n=1}^{M} \left(x_n - A\cos(\omega t_n) - B\sin(\omega t_n) - C\right)^2 \tag{1}$$

If the frequency ω is known, the model is linear in the parameters, so *linear least squares* is performed (three-parameter method), which can be calculated directly (numerically, after factorisation of the system matrix). If the frequency is unknown, *nonlinear least squares is executed* (four-parameter method), a method iteratively determining the parameters of the sine wave.

Least squares fit has very nice properties, especially when the error sequence (the difference between the samples and the model) is random, zero-mean Gaussian and the samples are independent. In this case the linear LS fit is unbiased, and has minimum variance. Nonlinear LS is asymptotically unbiased and has asymptotically minimum variance.

---

[1] In strict sense, the DC value is not a parameter of the sine wave, but it is a general parameter of the measured waveform. Offset and drift, ubiquitous in measurements, need to be modelled, and the DC value is a good tool to do this.

The LS fit exists and is reasonable even in cases when the above assumptions concerning the error sequence are not met, but the properties of the estimates of the parameters may not be as nice as above. In the testing of ADC's, this is definitely the case: the ideal quantization error and the ADC nonlinearity error often dominate over observation noise, therefore they will also dominate the properties of the results. LS fitting still works, as it usually works, especially because the error values are more or less scattered, but since the error is deterministically related to the input and to the transfer characteristic of the ADC, the result is prone to possible strange errors. Because of the non-idealities, the fit of the input sine is not perfect, and the error of this fit is carried further in the analysis of the residuals. Therefore, optimal fit, even if the errors deviate from the above assumptions, is critical. The error is not very large, but if we consider that even small deviations of the sine parameters cause relatively large deviations in the residuals, the importance of determining the achievable 'best' sine fit is clear.

In the IEEE standards [1-2] the above problem is not mentioned. They simply assume that the LS fit works well. Practitioners already observed the phenomenon of the uncertainty in the determination of the ENOB, or of the equivalent resolution, but up to now no systematic treatment was suggested.

An improvement of the fit has already been suggested and analysed in [6]. The basic idea will be illustrated here with the help of the plots of the error, given for an ideal ADC characteristic.

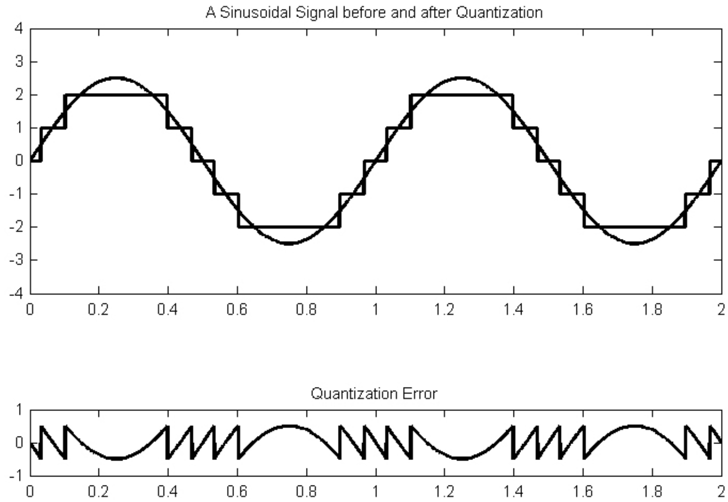

Fig. 1 Quantization of a sine wave in an ideal quantizer, with the dc offset is zero

It is clear from the figure that the error sequence is deterministic, with definite patterns.

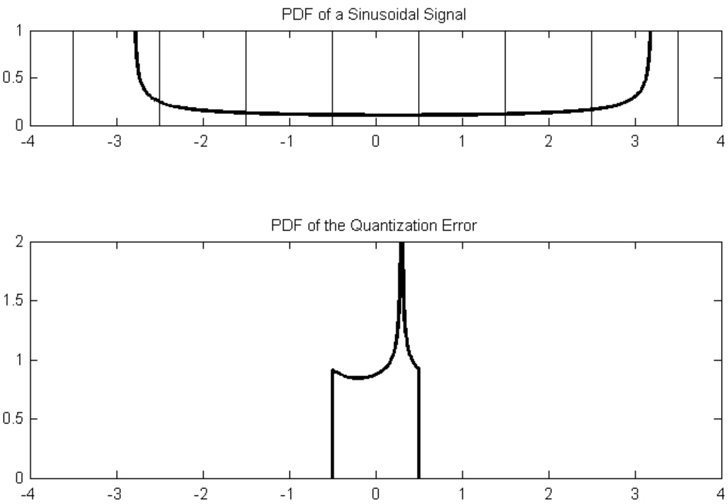

Fig. 2 Probability density function of a sinusoidal signal (amplitude: 3Δ), and that of the quantization error. The dc offset is dc = 0.2.

If the conversion noise of the ADC is small, the ADC error is deterministic, with many of the error samples occurring around the sine peak values. If the sine is considered having random phase, the ADC error will have high probability to be around the corresponding values. From Fig. 2 it is clear that the probability density function of the error is not uniform. Since the DC value and the amplitude of the sine wave both influence the position of the peak in the PDF of the error, and minimization of Eq. (1) is equivalent to the minimization of the variance of the error, the estimated parameters are always those which tend to bring the peak in Fig. 2 to the centre.

A good solution to this problem, suggested in [6], is to eliminate the samples in the ‘pathological’ (not totally filled) bins from the fit. By this, the distortion in the parameters can be significantly decreased.

It turns out that after this manipulation, the error in the fit can be further decreased, and by this, residual analysis improved. To do this, analysis of the error is necessary.

II. Analysis of the quantization error

In order to understand what happens with the remaining samples, let us plot the PDF of the sine wave in an ideal quantizer, along with the quantization levels.

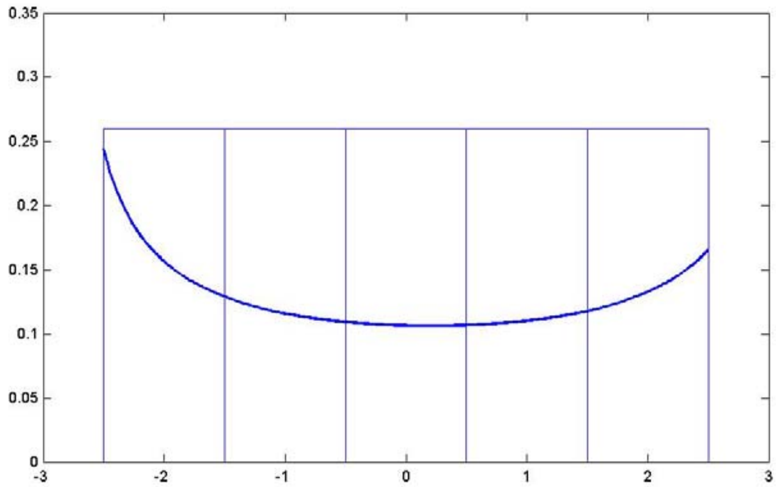

Fig. 3 Quantization of the considered samples

We can observe that the ‘not-pathologic’ samples exhibit a given pattern. The PDF of the sine wave,

$$f(x) = \frac{1}{\pi\sqrt{A^2 - (x-\mu)^2}}, \tag{2}$$

is a function monotonically increasing towards both amplitudes. Therefore, the probability of errors is always larger towards the peaks than towards the mean value. The more probable values are all negative (the quantized value is below the true value towards the peaks). Therefore, when minimizing Eq. (1), the ‘outside’ samples with negative errors will have more weight. This means ‘for the algorithm’ that a slightly smaller sine amplitude provides a smaller cost function, that is, the optimal amplitude is slightly underestimated. This is illustrated in Fig. 4.

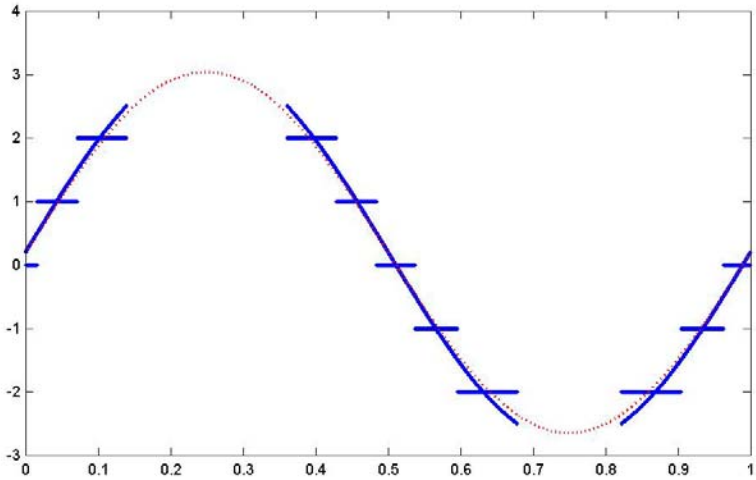

Fig. 4 Fitting the samples of a quantized sine wave: the fitted sine (dotted line) has smaller amplitude than the original one.

III. Weighted Least Squared to improve fitting

How can this systematic fitting error be avoided? If we realize that the cause is the asymmetric distribution of the errors, we can correct for this asymmetry, at least when knowledge of the distribution of the sine wave is at hand. We can use the *reciprocal* of the PDF in Fig. 3 as a weighting function.

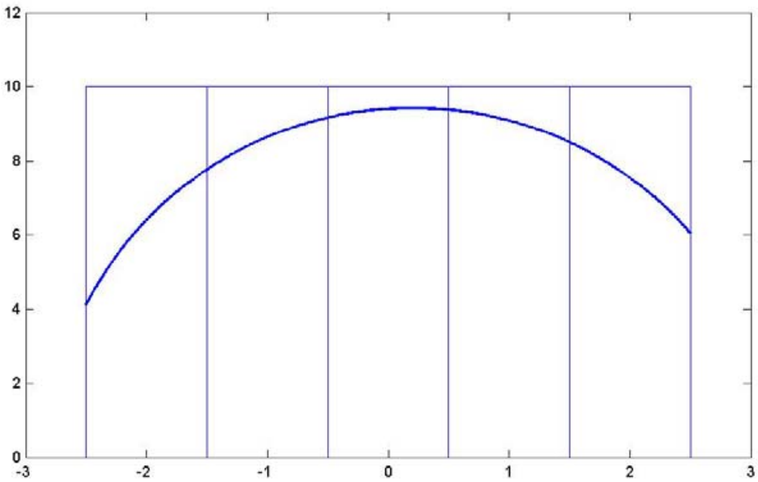

Fig. 5 Weighting by the reciprocal of the probability density function

With this weighting, estimate of the amplitude, which was $A = 2.84$ until now, has been improved to $A = 2.91$. The true value is 3.

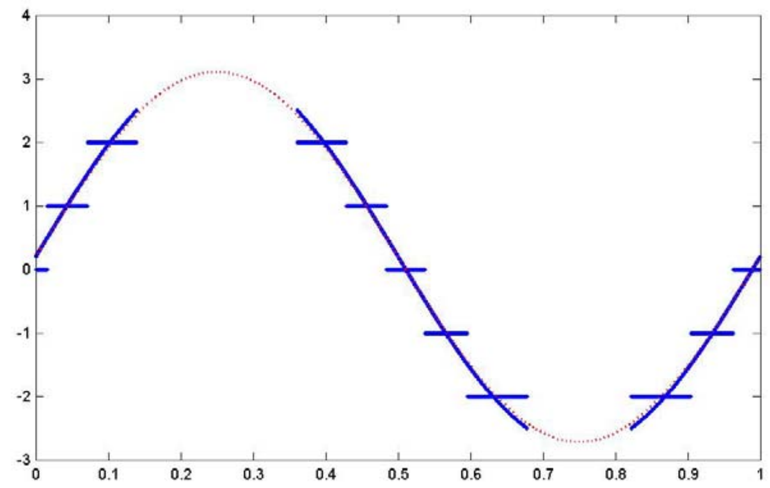

Fig. 6 Improved fitting the samples of a quantized sine wave with weighting by the reciprocal of the PDF of the sine wave

This weighting is good for the ideal quantizer, but when there is some noise involved, the central part is seemingly overemphasized. The variance of the noise is usually independent of the instantaneous value of the sine wave, therefore optimal estimation of the amplitude involves equal weights of the noise samples. This is in contradiction to the above requirement. A compromise could be reached if the *global* weighting remained constant, while within each bin weighting was changed according to the requirement that in a bin the distribution of the samples is approximately constant. This can be achieved by the following weighting.

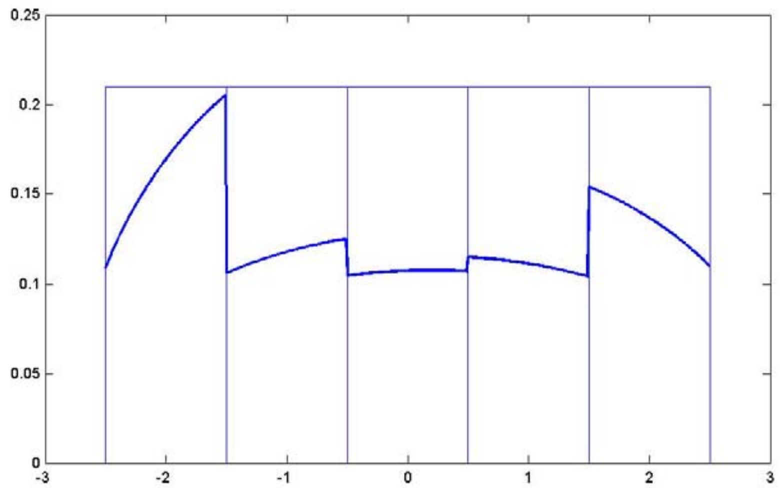

Fig. 7 Weighting while maintaining the probabilities in each bin

The fitted sine amplitude is 2.97, very close to the true 3.00.

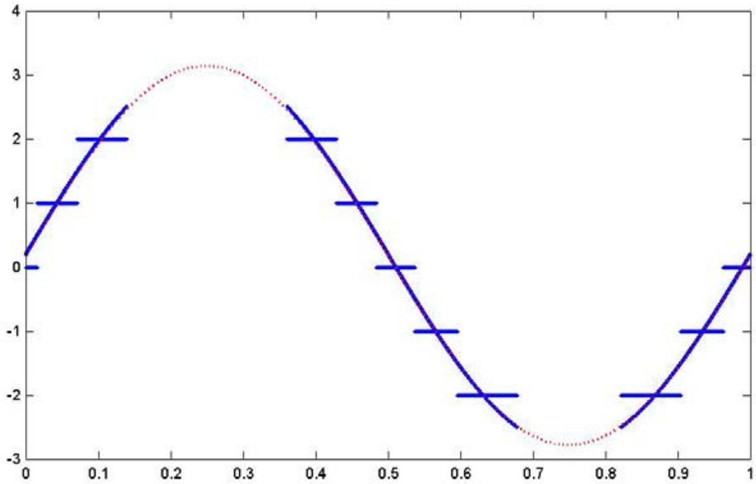

Fig. 8 Improved fitting the samples of a quantized sine wave with weighting by the reciprocal of the PDF of the sine wave, maintaining the probability in each bin

IV. The Algorithm

The above algorithm can only be applied when the probability belonging to any bin, and the PDF of the samples, are both known. This is only possible when the parameters of the sine wave are known. Fortunately, among the bins, and also within the bins, the weighting is smooth, so an *approximate* knowledge is sufficient. Therefore the following iterative procedure can be suggested:

1. eliminate the ‘pathological’ bins, that is, samples which are at the maximum histogram bins or outside these [6],
2. make an estimation of the amplitude of the sine wave, by an LS fit (3- or 4-parameter method),
3. determine the weighting corresponding to Fig. 7,
4. make WLS fit with these weights,
5. return to 3 once if necessary.

V. Conclusions

Besides the effect of the partly filled bins around the sine peaks, a secondary source of fitting errors (that is, a source of error in the residual analysis, like ENOB calculation) was identified. A modified algorithm was suggested, which eliminates this error, without deteriorating the properties of the analysis otherwise.

References

[1] IEEE Standard 1241-2000, “IEEE Standard for Terminology and Test Methods for Analog-to-Digital Converters.”
[2] IEEE Standard 1057-1994, “IEEE Standard Digitizing Waveform Recorders.”
[3] J. J. Blair, “Sine-fitting Software for IEEE Standards 1057 and 1241,” Proc. IEEE Instrumentation and Measurement Technology Conference, IMTC/1999, Venice, May 24-26, 1999. pp. 1504-6.
[4] I. Kollár and J. Márkus, “Standard Environment for the Sine Wave Test of ADC’s.” Measurement, Vol. 31, No. 4, pp. 261-69, 2002.
[5] T. Z. Bilau, T. Megyeri, A. Sárhegyi, J. Márkus, and I. Kollár, “Four Parameter Fitting of Sine Wave Testing Results: Iteration and Convergence,” Computer Standards and Interfaces, Vol. 25, No. 2, 2003, pp.
[6] Kollár, I. and J. J. Blair, “Improved Determination of the Best Fitting Sine Wave in ADC Testing,” IMTC/2004, Proc. IEEE Instrumentation and Measurement Technology Conference, IMTC/2004, Como, May 20-22, 2004. pp. 829-34.