Analysis of a Multiplying DAC Employed for Synchronous Detection

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Abstract- Multiplying digital to analog converters can be used as synchronous detection circuits by combining a proper code sequence and an analog input at the reference. The output for a periodic signal has been derived under reasonable assumptions. The analysis shows that this type of synchronous detector is sensible only to harmonic components almost equal to a multiple of the number of samples per period. This result can be seen as a generalization of the classical two-level synchronous detection theory.

I. Introduction

The multiplying digital to analog converter (MDAC) is an electronic component frequently used for electronic constructions. In comparison with normal digital to analog converter (DAC), in a MDAC the external reference can generally operate in a wider range and in a wider frequency band. On the other hand, for the specific circuits employed for its construction, the resolution is generally limited to 14-16 bits. This component is utilized for building many types of electronic circuits such as, for example, adjustable generators and special control systems.
A schematic representation of the device is given in Fig. 1.

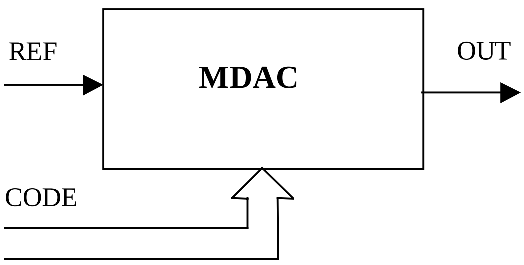

Figure 1 Schematic representation of a multiplying digital to analog converter (MDAC)

The MDAC can be thought as a hybrid multiplier, where the two multiplicands are the REF input, which can be a voltage or a current signal and the CODE input, which is a set of digital signals. The result output OUT is an analog signal (voltage or current) whose instantaneous value *O(t)* is given by the product of reference signal *Ref(t)* multiplied by the analog equivalent of the code $\{C_i(t)\}$.

$$O(t) = Ref(t) \cdot An_{eq}(\{C_i(t)\}) \tag{1}$$

For its particular characteristics the MDAC is generally employed, in the field of precision measurements, as a subsystem for impedance measurements or in system for voltage, power and energy measurements. A further application of this component, a bit outside of the usual ones, is for synchronous detection [1], where it can be used both for replacing expensive lock-in amplifiers and for synchronous filter. The best utilization is for circuits where there are other circuits digitally driven and it is easier to produce the suitable codes for the modulation or the demodulation.
The present paper analyses the particular features of such a device for a more appropriate application of the component for this purpose.

II. The synchronous detection

The synchronous detection is the extraction of the harmonic components from a period signal by means of a combination with a reference signal. In case of a MDAC the reference signal can be a virtual sinusoidal signal given by a proper code sequence. The best approximation of a sinusoidal signal is a staircase waveform, where all steps have the value of the central point of a sinusoid [2]. As a first approximation it is assumed that the amplitude has infinite resolution. The voltage values of the *N* steps in one period, apart from a possible gain factor, are then given by the relation:

$$st(t)\Bigg|_{\left(i-\frac{1}{2}\right)\frac{2\pi}{\omega N}}^{\left(i+\frac{1}{2}\right)\frac{2\pi}{\omega N}} = S_i = \sin\left(i\frac{2\pi}{N}\right) \tag{2}$$

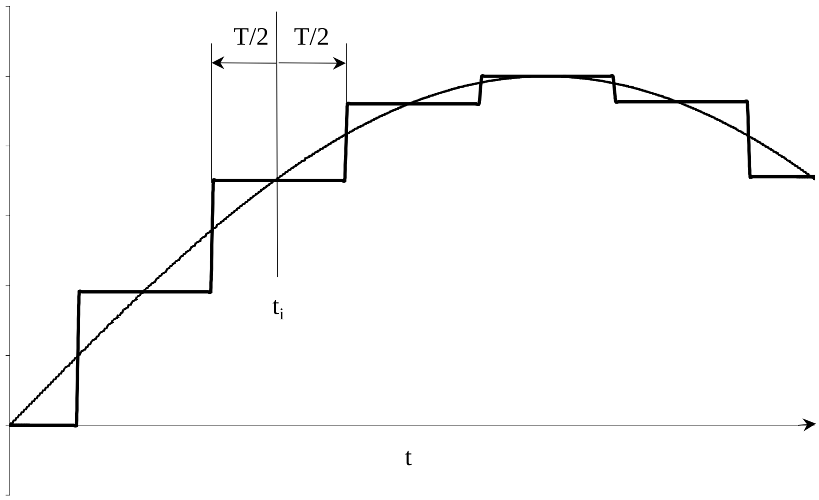

Figure 2 Staircase approximating a sinusoidal signal, T is the sample period.

By applying a periodic signal *ps(t)* of the same frequency of the staircase,

$$ps(t) = \sum_{m=1}^{H}\left(A_m \cos\left(m\cdot\omega\cdot t + \frac{2\pi\cdot m}{N}\right) + B_m \sin\left(m\cdot\omega\cdot t + \frac{2\pi\cdot m}{N}\right)\right) \tag{3}$$

where A_m and B_m are respectively the cosine and sine components of the *m*-th harmonic of the signal and H the number of harmonics, the mean value at the output of the MDAC produced by *m*-th harmonic component will be:

$$M_m = \frac{1}{N\cdot T}\sum_{i=0}^{N-1} S_i\left[\int_{-\frac{T}{2}}^{+\frac{T}{2}} A_m \cos\left(m\cdot\omega\cdot\left(t+\frac{2\pi\cdot i}{\omega\cdot N}\right)\right) + B_m \sin\left(m\cdot\omega\cdot\left(t+\frac{2\pi\cdot i}{\omega\cdot N}\right)\right)\right] \tag{4}$$

where $T=2\pi/\omega N$ is the sample period. Then, by computing relation (4) the mean value due to the *m*-th harmonic component can be evaluated as:

$$M_m = \frac{\sin\left(\frac{m\pi}{N}\right)}{\left(\frac{m\pi}{N}\right)}\cdot R_m = \mathrm{sinc}\left(\frac{m\pi}{N}\right)\cdot R_m \tag{5}$$

where:

$$R_m = \frac{1}{N}\frac{\cos\left(\frac{\pi}{N}\right) sin\left(\frac{\pi}{N}\right)\cdot\left[A_m\cdot(1+\cos(m\pi))\cdot(1-\cos(m\pi)) - B_m\cdot\sin(m\pi)\cos(m\pi)\right]}{\left[\cos\left(m\frac{\pi}{N}\right)^2 - \cos\left(\frac{\pi}{N}\right)^2\right]} \tag{6}$$

By computing from relation (6) the R_m values for the different harmonics, we can obtain values generally zero because the numerator is always zero. For $m=k\cdot N-1$ and $m=k\cdot N+1$ with $k\in\{\text{integer}\geq 0\}$, where also the denominator is zero, the result can be evaluated by differentiating both the numerator and the denominator and is given by:

$$R_m = \left\{ \frac{-B_m}{2} \quad for \quad m = k\cdot N - 1, \quad \frac{B_m}{2} \quad for \quad m = k\cdot N + 1, \quad 0 \quad for \quad other \quad values \right\} \tag{7}$$

For such values of m the $\mathrm{sinc}(m\pi / N)$ are:

$$sinc\left(\frac{m\pi}{N}\right) = \left\{ \frac{-\cos(k\cdot\pi)}{k\cdot N - 1}\mathrm{sinc}\left(\frac{\pi}{N}\right) \quad for \quad m = k\cdot N - 1, \quad \frac{\cos(k\cdot\pi)}{k\cdot N + 1}\mathrm{sinc}\left(\frac{\pi}{N}\right) \quad for \quad m = k\cdot N + 1, \right\} \tag{8}$$

By combining the results of (7) and (8) it is possible to conclude that there are only $k\cdot N-1$ or $k\cdot N+1$ harmonics, where $k\in\{\text{integer}\geq 0\}$, and furthermore for these values the ratio between the amplitudes of the m-th harmonic and the fundamental is in theory exactly equal to $1/m$.

$$M_m = \left\{ \frac{B_m}{2}\cdot\frac{(-1)^k}{k\cdot N \pm 1}\mathrm{sinc}\left(\frac{\pi}{N}\right) \quad for \quad m = k\cdot N \pm 1 \right\} \tag{9}$$

Then, the sensitivity to the harmonics of the mean value at the output of the MDAC, when the code supplied to the digital side approximates a sinusoidal signal, is represented in the graph of Fig. 3.

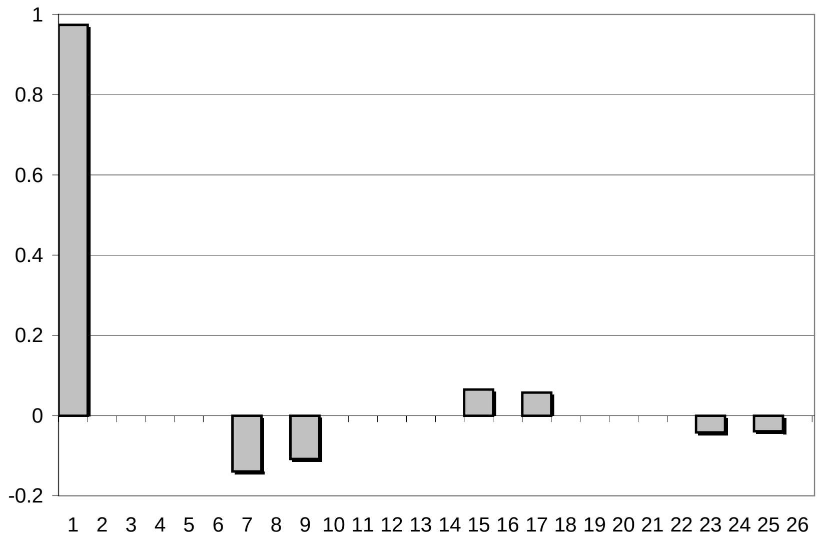

Figure 3 Sensitivity to harmonic components in the reference signal when the input code approximates a sinusoidal signal by means of a staircase of N=8 step per period.

Similarly, by applying the same approach using a cosine approximation for the staircase:

$$st(t)\Bigg|_{\left(i-\frac{1}{2}\right)\frac{2\pi}{\omega N}}^{\left(i+\frac{1}{2}\right)\frac{2\pi}{\omega N}} = S_i = \cos\left(i\,\frac{2\pi}{N}\right) \tag{10}$$

The sensitivity to harmonic can be evaluated analytically as:

$$M_m = \left\{\frac{A_m}{2}\cdot\frac{\pm 1\cdot(-1)^k}{k\cdot N\pm 1}\operatorname{sinc}\left(\frac{\pi}{N}\right) \qquad \mathit{for} \quad m = k\cdot N\pm 1\right\} \tag{11}$$

In case of N=2, relation (11) can be seen as the sensitivity to harmonics of the classical two-state synchronous demodulator.

III. Conclusions

The use of multiplying digital to analog converters for synchronous detection circuits has been analyzed for its response to harmonics. The output for a periodic signal shows that this device is sensible only to harmonic components equal to a multiple +1 o -1 of the number of samples per period. In this case the sensitivity is proportional to the reciprocal of the harmonic order. This result is a generalization of the classical two-level synchronous detection theory.

References

[1] U. Pogliano, F. Cabiati, "The multiplying digital to analog converter as a synchronous detector", *Digest of the Fourth British Electromagnetic Measurements Conference*, Teddington (UK), 7-9 November 1989, pp. 13/1-13/3

[2] U. Pogliano, “Precision Measurement of AC Voltage below 20 Hz at IEN”, *IEEE Trans. on Instrum. and Measur.*, vol. IM-46, no. 2, pp. 396-372, 1997