# Measurement and Representation of the ADC Integral Nonlinearity in the Time Domain

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***Abstract--*** In this paper a new approach to the measurement and compensation of the *soft* components of the integral nonlinearity (INL) of the ADC in the time domain is presented, using both sine and triangular waves. Connection will be established between the user oriented nonlinear description based on memoryless nonlinear models using orthogonal polynomial series and the manufacturer oriented specification, the INL. Using the orthogonal series representation it will also be shown how the nonlinearity of the generator can be compensated *a posteriori* in the ADC characterization, allowing for a better assessment of the ADC. Finally, simulation results will be presented that portray the applicability of these methods and representations.

## I. Introduction

One of the most important assessments needed to be carried out when characterizing an ADC is its nonlinear behaviour. This goal is usually achieved either through the measurement of the stationary integral nonlinearity vector (INL), by assessing the harmonic distortion at a given frequency, or evaluating the two-tone intermodulation coefficients. The knowledge of the first one has traditionally been related to the histogram method, the second with both the time and frequency domain tests while the third one only with the frequency domain test. The assessment of the converter's INL has the obvious advantage of allowing for the output correction of the ADC since it carries the knowledge of its stationary transfer characteristic (*tc*) [1].

An exception to the requirement of the histogram method for the assessment of the INL has been the work in [2] where the low order components or *soft* representation of the INL has been accomplished through parametric estimation of the stimulus spectra and use of the Chebyshev orthogonal polynomial series representation. The measurement of the INL resulting from the histogram test has higher resolution than the one emerging from the frequency domain tests but at the cost of much higher characterization time. The histogram test is also more robust than the frequency domain tests since it is much less sensitive to overvoltage and noise, yet again at the expense of even higher characterization times. The histogram test, however, has the appeal of allowing for the use of stimulus other than the sine wave, namely the triangular wave and gaussian noise. The method presented in this paper, which allows for the evaluation of the INL from the time domain representation of the stimulus, is more robust than the frequency domain test and allows for the use of any deterministic stimulus, as long as it exhibits half-wave symmetry, while requiring roughly the same number of samples as the frequency domain test.

From the user perspective, the knowledge of the INL is not as useful as it should be since it does not directly relate to the nonlinear model of any other electronic device. A much better approach would be the use of a higher level nonlinear model, useable in all devices. That goal can be achieved through the use of orthogonal polynomial series which can model or account for memoryless nonlinearities. In section II we will present the analytical relation between the INL and the orthogonal polynomial representation, which enables us to also establish the connection between the shape of the INL and the order of the distortion it imposes.

One advantage of levelling the nonlinear models amongst electronic devices, which we will explore in this paper, is the possibility to compensate, *a posteriori*, the nonlinearities of the generator. Since we will be considering the measurement and representation of the INL in the time domain, this will be the domain where we will show its application. In [3,4] we have shown how the compensation of the generators distortion in the stochastic domain could be attained.

## II. Memoryless representation of nonlinearities

The Weierstrass approximation theorem [5] states that a continuous and limited input-output memoryless *tc* of any nonlinear device can be described, to any desired accuracy, by means of a polynomial series. Naturally, the *tc* of an ADC is not continuous and the approximation will never be perfect using a polynomial series. However, it can be used to describe the trend associated with a given ADC *tc*, and therefore its nonlinearity, usually termed *soft* nonlinearity which is continuous.

To assess the nonlinearity we will assume the input/output characteristic can be expressed by the nonlinear equation,

$$y = \mathrm{f}(x), \tag{1}$$

where both the stimulus, *x*, and the output, *y*, are functions of time. This will be considered to be the direct system, relating the output with the input.

To model the memoryless nonlinearities of the signal we will make use of a generalized Fourier series expansion considering as base a given set of orthogonal polynomials, $\{\phi_k(x)\}$,

$$y = \mathrm{f}(x) = \sum_{k=0}^{\infty} \breve{h}_k \phi_k(x). \tag{2}$$

The choice of the orthogonal polynomial family is naturally a result of the weighting function – *pdf* of the stimulus *x* – to which the family is orthogonal, *i.e.*,

$$\int \phi_m(x)\phi_n^*(x) f_x(x)\, dx = \begin{cases} \sigma_n^2 \Leftarrow m = n \\ 0 \Leftarrow m \neq n \end{cases}, \tag{3}$$

where $f_x(x)$ corresponds to the stimulus *pdf* and $\sigma_n^2$ the mean squared value of the n$^{th}$ component of the base. For three common stimuli, the sine wave, the triangular wave and gaussian white noise, the orthogonal families are the Chebyshev polynomials of the first kind, the Legendre polynomials and the Hermite polynomials, respectively. The $\breve{h}_k$ in (2) are the weights of the base components and correspond to a measure of the representativeness (cross-correlation) of the n$^{th}$ order distortion of the signal. The inverse system, relating the input with the output, can also be described by means of a polynomial representation,

$$x = \mathrm{f}^{-1}(y) = \sum_{k=0}^{s} \breve{h}_k \phi_k(y). \tag{4}$$

To invert an orthogonal polynomial representation we will consider the broader case of system composition – of which the inverse is a particularization. Let us consider three systems with orthogonal descriptions,

$$\mathrm{f}(x) = \sum_{k=0}^{s} \breve{f}_k \phi_k(x);\ \ \mathrm{g}(v) = \sum_{k=0}^{p} \breve{g}_k \phi_k(v);\ \ \mathrm{w}(x) = \sum_{k=0}^{s} \breve{w}_k \phi_k(x). \tag{5}$$

It can be shown that the composite system,

$$\mathrm{w}(x) = \mathrm{g} \circ \mathrm{f}(x), \tag{6}$$

can easily be determined through (orthogonal weight vectors), [4,6]

$$\breve{\mathbf{w}} = \breve{\mathbf{F}} \cdot \breve{\mathbf{g}},\ \ \breve{\mathbf{F}} = \left(\boldsymbol{\psi}\mathbf{Q}_{\mathrm{w}}\mathbf{F}\mathbf{Q}_{\mathrm{y}}^{-1}\boldsymbol{\psi}^{-1}\right), \tag{7}$$

where $\boldsymbol{\psi}$ is the conversion matrix between orthogonal series and power series representation for a given orthogonal family, $\mathbf{Q}$ is the normalization matrix and $\mathbf{F}$ is a matrix defined through,

$$\mathbf{F} = \begin{bmatrix} \mathbf{f}^{(*0)} & \mathbf{f}^{(*1)} & \cdots & \mathbf{f}^{(*r)} \end{bmatrix}, \tag{8}$$

$$\mathbf{f}^{(*n)} = \begin{bmatrix} 1 & \mathrm{f}_1^{(*n)} & \cdots & \mathrm{f}_{s\cdot n}^{(*n)} & \mathbf{0}_{(1,s(r-n))} \end{bmatrix}^T, \tag{9}$$

and [7],

$$\mathrm{f}_k^{(*n)} = \begin{cases} 1 & \Leftarrow\ k = 0 \\ \displaystyle\sum_{l=1}^{k}\left(\left(\frac{n+1}{k}\right)l - 1\right)\mathrm{f}_l\, \mathrm{f}_{k-l}^{(*n)} & \Leftarrow\ k > 0 \end{cases}. \tag{10}$$

The conversion matrix $\boldsymbol{\psi}$ referred to in (7) can be built by making the n$^{th}$ column correspond the orthogonal description of $x^n$, *i.e.*,

$$\boldsymbol{\psi}_n \to \psi_{0n}\phi_0(x) + \cdots + \psi_{nn}\phi_n(x) = x^n, \tag{11}$$

which can be numerically derived through proper expansion of the equations in TABLE 1. In (7), $\mathbf{Q}$ is a normalization matrix required to guarantee the argument of the orthogonal polynomials remains within it's orthogonality domain. The nth column of $\mathbf{Q}$, $\mathbf{Q}_n$, corresponds to the coefficients of the polynomial,

$$Q_n \to \left(Av + O\right)^n, \tag{12}$$

where *A* is the stimulus amplitude or standard deviation, *O* its offset or mean, and *v* just a dummy variable. According to (7) we need also their inverses and not necessarily referred to the same variables, at least in the case of the normalization matrix.

TABLE 1
EXPRESSIONS TO OBTAIN ELEMENTS OF MATRIX, $\boldsymbol{\psi}$

| | | |
|---|---|---|
| $x^n = \frac{n!}{2^{n/2}} \sum_{k=0}^{[n/2]} \frac{1}{k!(n-2k)!} H_{n-2k}(x/\sqrt{2})$ | $-\infty < x < \infty$ | [8] |
| $x^n = \frac{1}{2^{n-1}} \sum_{k=0}^{[n/2]} \binom{n}{k} T_{n-2k}(x) - \frac{1}{2^n}$ | $-1 \le x \le 1$ | [9] |
| $x^n = \frac{n!}{2^n} \sum_{k=0}^{[n/2]} \frac{n+1/2-2k}{k!\left(\frac{1}{2}\right)_{n+1-k}} P_{n-2k}(x)$ | $-1 \le x \le 1$ | [8] |

The inverse of the normalization matrix can be constructed by filling the columns of $\mathbf{Q}^{-1}$ with the coefficients of the power series,

$$\mathbf{Q}_n^{-1} \to \left(\frac{v-O}{A}\right)^n, \tag{13}$$

and the columns of $\boldsymbol{\psi}^{-1}$, $\boldsymbol{\psi}_n^{-1}$, correspond to the decomposition of $\phi_n(x)$ into a power series,

$$\boldsymbol{\psi}_n^{-1} \to \psi_{0n}^{-1} x^0 + \cdots + \psi_{nn}^{-1} x^n = \phi_n(x). \tag{14}$$

The coefficients, $\psi_{m,n}^{-1}$, can be numerically derived using TABLE 2 together with the following recurrent equation [10],

$$a_{1n}\phi_{n+1}(x) = a_{2n} x \phi_n(x) - a_{3n}\phi_{n-1}(x) \tag{15}$$

TABLE 2
COEFFICIENTS CONNECTED WITH THE RECURRENT EQUATION (15)

| $\phi_n(x)$ | $a_{1n}$ | $a_{2n}$ | $a_{3n}$ | $\phi_0(x)$ | $\phi_1(x)$ |
|---|---|---|---|---|---|
| $H_n(x/\sqrt{2})$ | 1 | $\sqrt{2}$ | $2n$ | 1 | $\sqrt{2}x$ |
| $T_n(x)$ | 1 | 2 | 1 | 1 | $x$ |
| $P_n(x)$ | $n+1$ | $2n+1$ | $n$ | 1 | $x$ |

Inversion corresponds to a particularization of (6) in which $\mathrm{w}(x) \equiv x$ and, therefore, we intend to find a $\mathrm{g}(v)$ such that,

$$x = \mathrm{g} \circ \mathrm{f}(x) \tag{16}$$

from the known $\mathrm{f}(x)$. Due to the need for normalization of the orthogonal polynomial arguments, to find the inverse polynomial description (7) can be rewritten as,

$$\breve{\mathbf{g}} = \left(\boldsymbol{\psi}\mathbf{Q}_{\mathbf{x}}\mathbf{F}\mathbf{Q}_{\mathbf{v}}^{-1}\boldsymbol{\psi}^{-1}\right)^{-1} [O_x \quad A_x \quad 0 \ \cdots \ 0]^T, \tag{17}$$

where $O_x$ and $A_x$ are the offset and the amplitude of the stimulus, respectively.

So far we have been able to relate the time domain input/output characteristic of any continuous nonlinear device. In the case of the ADC the output is discrete and the device possesses discontinuities which means that both (2) and (4) will only be valid as an approximation, albeit a very good one if enough distortion terms are considered. However, the bridge must still be crossed between the orthogonal polynomial description and the usual ADC nonlinearity metric, the INL.

According to the definition [11], the INL is a function of the code of the converter, that is, of the output, while the nonlinear model of (2) is a function of the input. This is empirically represented in Fig. 1 where both the direct and inverse nonlinear description are shown.

Besides the inverted relation between the two nonlinear descriptions the INL, by definition, specifically accounts for the deviations in the code transition levels, relative to the ideal converter with the same resolution and range. It is simple to prove that,

$$T_e[i] = \sum_{k=0}^{\infty} \breve{\mathrm{f}}_k^{-1} \cdot \phi_k\left(\frac{T_t[i]-O}{A}\right), \tag{18}$$

where $T_e[i]$ represents the experimental lower transition level associated with code $i$, $T_t[i]$ the theoretical counterpart and $A$ and $O$ correspond to the amplitude and offset of the converter, respectively. Equation (18) allows for the determination of the transfer characteristic of the converter from which the INL can be obtained directly either considering the terminal based definition or a least square error criterion.

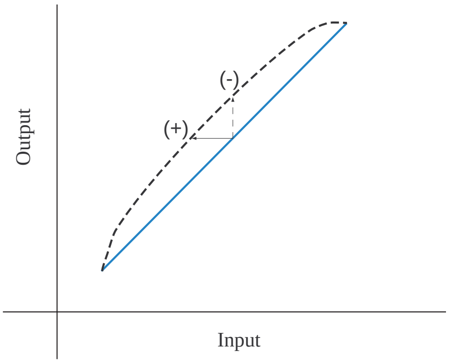


Fig. 1 Representation of the nonlinear behavior accounted for by the INL [⟷] and by the polynomial model of (2) [⁝].

### III. The ADC time-domain characterization

For the characterization of the ADC, according to what we have seen in the previous section, we need only to measure the polynomial coefficients in (18). In [3,4] we have shown how they can be measured from the histogram method and in [2] is shown how to derive them from the spectral domain. In this paper we will show how to derive them from the time-domain, but we will go a step further and take advantage of the ease with which one can algebraically manipulate polynomial nonlinear models to compensate for the nonlinearities of the generator – the characterization of a nonlinear device considering a distorted stimulus. To this end we will consider a setup similar to the one depicted in Fig. **2**.

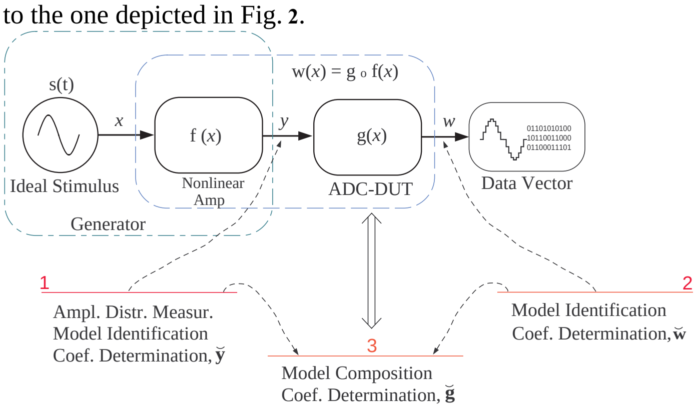


Fig. 2 Block diagram of setup associated with the characterization of a nonlinear device using a distorted waveform.

The steps portrayed in this figure depict the need for two nonlinear model assessments: one analog, related to the distortion of the generator [step 1], and the second one digital, related to the composite distortion of generator and acquisition channel [step 2]. The third and final step corresponds to the algebraic composition of the two models to identify the nonlinear model of the DUT. The first step, assessment of f($x$), can be accomplished by means of the method proposed in [3,4] while the second step, assessment of w($x$), can be accomplished through time-domain fitting of w($x$) using an ideal model for $x$.

The characterization in the time domain is usually done by fitting a sine wave and from the residuals of the best fit to assess the effective number of bits (ENOB) and the signal to noise plus distortion ratio (SINAD) [11-14]. Here we will adopt an extension to the conventional procedure which consists in the following steps,

1. Acquire either a sine wave or a triangular wave with more than 10 periods of the signal and at least 10 points per period.
2. Use interpolated FFT (IpFFT) analysis of the data vector to get an accurate estimate of the frequency of the first harmonic.
3. Based on this guess trim the data vector to contemplate an approximately integer number of periods.
4. Use this frequency estimate as a starting point to an iterative 4-parameter sine or triangular fit. Here we prefer the use of a Gauss-Newton approach that makes use of all four incremental parameters instead of the method proposed in the standard [11] which contemplates only the frequency [15].

After these four steps the undistorted version of the stimulus has been reconstructed. To find the weights of each component of the orthogonal base we will follow a procedure made up of the following steps,

1. From the original data vector subtract the reconstructed ideal stimulus obtaining the vector of residues.
2. From the vector of the reconstructed ideal stimulus, forcing unitary amplitude and null offset, create a vector with the $n^{th}$ order distortion by applying it to the base function, $\phi_n(x)$.
3. Best fit this vector to the vector of residues and find the corresponding coefficient.
4. Subtract the newly found nth order distortion component from the vector of residues and repeat steps 1 to 4 until the required number of distortion terms has been found.

After these two procedures the identification of the composite nonlinear model, $\breve{\mathbf{w}}$, has been found. The user oriented memoryless nonlinear formulation of the ADC can now be found through [see (7)],

$$\breve{\mathbf{g}} = \breve{\mathbf{F}}^{-1}\breve{\mathbf{w}}\,. \tag{19}$$

This description, due to its polynomial approach, can only account for *soft* nonlinearities. By properly inverting the *soft* nonlinear model of (19), the nonlinear behaviour of the ADC can be compensated. This, however, is not a very practical solution, as it requires the working signal to be the same as the stimulus used in the characterization. A better approach consists in inferring the integer nonlinearity vector (INL) of the ADC or acquisition channel.

## IV. Simulation results

In Fig. 3 we can see a simulated example of the proposed methodology for the assessment in the time domain of the nonlinear behavior of a 12 bit converter. The imposed INL, albeit only simulated, was purposely made close to what would be expectable from an ADC with SAR architecture. The simulated characterization was done using a sine wave, and a 120 term Chebyshev polynomial was used to describe the memoryless nonlinear behavior.

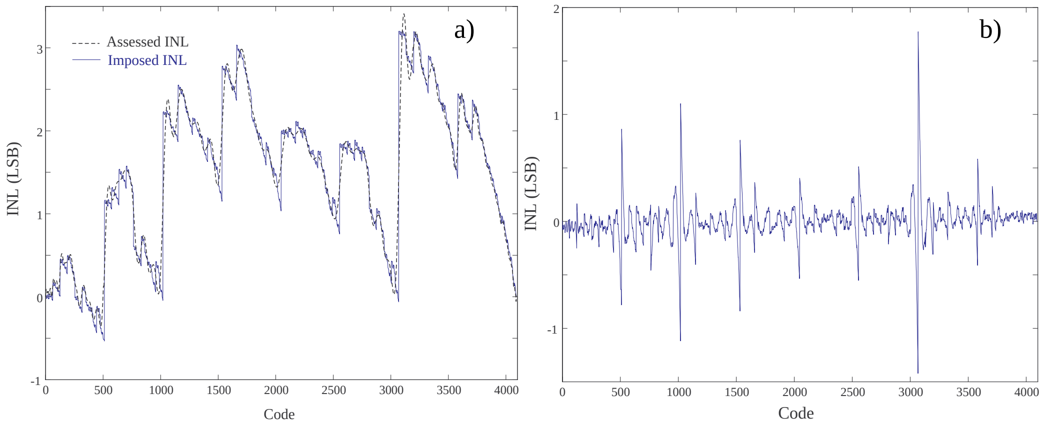


Fig. 3 **a)** Simulated assessment of the INL of a converter in the time domain – 120 term Chebyshev series. **b)** Difference between the imposed and assessed INL of a).

For the results depicted in Fig. 3a and Fig. 3b a sample vector of length $2^{14}$ was used. The ratio between the acquisition rate and the stimulus frequency was such as to allow the vector to contain 40 periods of the stimulus. The spikes that appear in Fig. 3b are a result of the limited *slew rate* of the fitted model considering its finite distortion order (120). As can be seen the trend of the INL is perfectly represented.

In Fig. 4a and Fig.4b, a simulated example is shown of the measurement of an ADC when the stimulus generator exhibits heavy distortion. In Fig. 4a it is shown both the imposed (theoretical) INL of the simulated 8-bit ADC and the composite distortion (generator+ADC) measured at the output of the ADC. After simulating the measurement of the distortion of the generator itself by means of the procedure laid out in [3,4], and identifying the composite nonlinear model using the time domain, the isolated nonlinear model of the ADC has been extracted and the corresponding INL assessed, as represented in Fig. 4b by the solid line. The stimulus used for this simulation was a triangular wave with $2^{15}$ samples spanning 52 complete periods. The memoryless nonlinear model was established by using a 50 term Legendre polynomial.

## V. Conclusion

We have presented an innovative method for modeling, compensation and extraction of the memoryless nonlinear behavior of a DUT using a distorted stimulus generator. This is accomplished by making use of orthogonal polynomials to model the nonlinear behavior of both the stimulus and the composite nonlinear system made up of the generator and the DUT. The composite model can be determined either by means of amplitude distribution of the output wave, in the case of an analog DUT, or through analysis of the data in the time domain, when in presence of acquisition channels. The results demonstrate the applicability of the method and, being simulations, exhibit very good agreement.

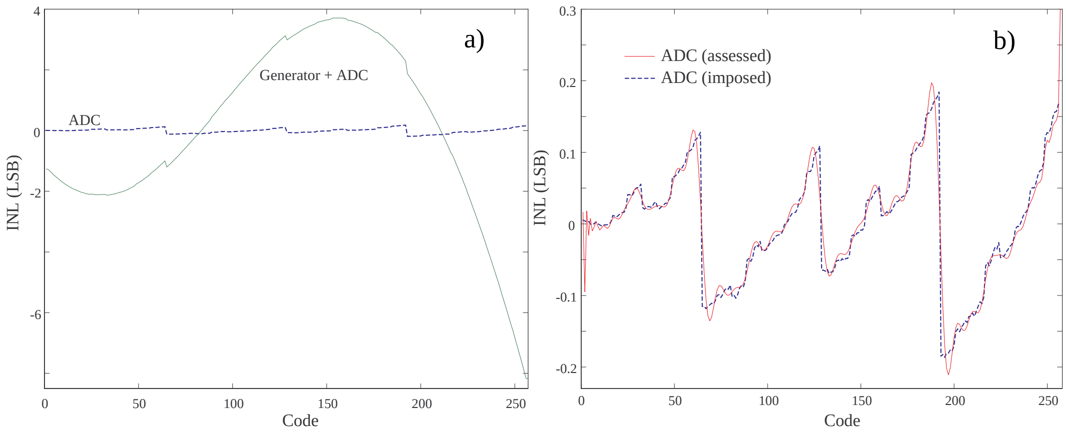


Fig. 4 **a)** INL identification of a simulated 8 bit ADC (dashed line) using the time domain test and an also simulated highly nonlinear generator whose equivalent INL is depicted by the solid line. **b)** The identified or assessed INL extracted from the composite distortion (Generator+ADC) using the methods presented in this paper are represented by the solid line whilst superimposed we find the compelled ADC INL against which it must match (dashed line). The assessed INL uses a 50 term Legendre polynomial approximation.

The possibility of compensating the distortion of the generator allows for the use of lower grade, therefore cheaper, generators, and to extend the bandwidth of the current nonlinear measuring capabilities. One further advantage of the method is its inherent ability to use stochastic signals.

The use of the time domain to perform the assessment of the nonlinear behavior exhibits two important properties: i) is extremely fast requiring few samples, and ii) is robust since it can withstand a slight saturation of the stimulus, guaranteeing the coverage of the full ADC range.

Finally, we have also seen how the ADC manufacturer’s nonlinear specification (INL) relates to the polynomial memoryless model of nonlinearities, better suited to a user oriented specification.

## References


[1] Giaquinto, N., Savino, M., and Trotta, A., "Detection, digital correction and global effect of A/D converters nonlinearities," *International Workshop on ADC Modelling and Testing - IWADC*, Smolenice Castle, Slovak Republic, pp. 122-127, 1996.

[2] Attivissimo, F., Giaquinto, N., and Kale, I., "Inl reconstruction of A/D converters via parametric spectral estimation," *Instrumentation and Measurement Technology Conference, 2003. IMTC '03. Proceedings of the 20th IEEE*, pp. 1166-1170, 2003.

[3] Martins, R. C. and Serra, A. C., “Representation and measurement of nonlinearities in stimulus signals,” *Instrumentation and Measurement, IEEE Transactions on*, vol. 52, no. 4, pp. 1160-1165, 2003.

[4] Martins, R. C. and Serra, A. C., "Stochastic Approach for Memoryless Nonlinearity Measurement," *International Workshop on ADC Modelling and Testing - IWADC*, Perugia, Italy, pp. 209-212, 2003.

[5] K. Weierstrass. *Über die analytische Darshell-bankeit sogenannter willkürlicher Funktionen reeler Argumente*, Math Werke, 1903.

[6] Tsimbinos, J. and Lever, K. V., “Nonlinear system compensation based on orthogonal polynomial,” *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, vol. 48, no. 4, pp. 406-417, 2001. 1057-7122.

[7] Donald E. Knuth. Seminumerical Algorithms. *The Art of Computer Programming*, Anonymous Addison-Wesley, 1969.pp. 444-451.

[8] Wolfram Koepf and Dieter Schmersau, “Representations of orthogonal polynomials,” *Journal of Computational and Appllied Mathematics*, vol. 90, pp. 59-96, 1998.

[9] William H. Press, Brian P. Flannery, Saul A. Teukolsky, and William T. Vetterling. *Numerical Recipes in C.* Cambridge University Press, 1997.

[10] Milton Abramowitz and Irene A. Stegun. *Handbook of Mathematical Functions*, New York: Dover

Publications, 1972.pp. 795-802.
[11] IEEE std 1271, Sep, 2001.
[12] Hejn, K. W. and Morling, R. C. S., “A semifixed frequency method for evaluating the effective resolution of A/D convertors,” *Instrumentation and Measurement, IEEE Transactions on*, vol. 41, no. 2, pp. 212-217, 1992. 0018-9456.
[13] Jenq, Y.-C. and Crosby, P. B., “Sinewave parameter estimation algorithm with application to waveform digitizer effective bits measurement,” *Instrumentation and Measurement, IEEE Transactions on*, vol. 37, no. 4, pp. 529-532, 1988. 0018-9456.
[14] Pintelon, R. and Schoukens, J., “An improved sine-wave fitting procedure for characterizing data acquisition channels,” *Instrumentation and Measurement, IEEE Transactions on*, vol. 45, no. 2, pp. 588-593, 1996. 0018-9456.
[15] Silva, M. F., Ramos, P. M., Martins, R. C. and Serra, A. C., "Least-Square Fitting Algorithms Applied To Periodic Signals," Proc. *IEEE Instrumentation and Measurement Technology Conf.* , Como , Italy , Vol. 3 , pp. 1730 - 1734 , May , 2004.