MODELLING OF INSTRUMENTATION SIGMA-DELTA ANALOG-TO-DIGITAL CONVERTERS

Valeriy I. Didenko, Andrey L. Movchan, Juriy S. Solodov

Department of Information and Measuring Technics,
Moscow Power Engineering Institute (Technical University),
Krasnokazarmennaia Street 14, 111250, Moscow, Russia.
Ph: +7 095 362 7368,
Fax: +7 095 362 7468,
E-mail: didenko@iit1.mpei.ac.ru

Abstract – The paper examines a behavioral model of sigma-delta analog-to-digital converters (SDADC) which allows the uncertainty of A/D conversion to be evaluated. The most sources of the uncertainty are represented in data sheets of modern SDADC. However, information given by manufacturers now is not enough to evaluate the conversion uncertainty with given probability. The most critical situation takes place for noise of SDADC – the major source of the uncertainty in many cases. This problem is mainly discussed in the paper. Conditions for evaluation of the conversion uncertainty with given probability are found from theoretical analysis, simulation of a typical SDADC electronic model and experimental data. Some propositions for including corresponding parameters and tests into standards are made.

Keywords: sigma-delta ADC, quantization error, conversion uncertainty.

1. INTRODUCTION

All analog-to-digital converters (ADC) can be roughly divided into instrumentation and multimedia types. The former devices are heart of Digital Multimeters, Digital Oscilloscopes and Data Acquisition Systems, the latter ADCs are used in Audio, Video, Television, Communications etc. According to [1], SDADCs are widely used for both instrumentation and multimedia applications. Performance of an instrumentation ADC is most fully described by the conversion uncertainty. This characteristic is mainly utilized in the same manner as used in metrology for the uncertainty of measurement [2], the result of the conversion being considered instead of a measurement result. The uncertainty is found in practice by worst case method or by means of probabilistic methods for given confidence probability. The worst combination of minimum and maximum values for each error constituents is used in the first case. Maximum value of a random quantity or random noise is offered to be evaluated as 3.3 times of the standard deviation [3]. For the normal distribution law this coefficient corresponds to the probability P≈0.999 that seems to be very close to P=1. For the second case, systematic errors are supposed to be random quantities for all devices of the same type [2]. Distribution law of these random quantities (not the random noise!) is usually supposed to be uniform (rectangular) one. Then the standard deviation of each constituent of the error is found as a difference between the maximum and the minimum values divided by $2\sqrt{3}$. The value of probability P=0.95 is usually used to evaluate the conversion uncertainty. The worst case method is sometimes described formally by P=1 [4].

The most important parameters of a multimedia ADC are signal-to-noise ratio (SNR), total harmonic distortion (THD) etc. [1]. Such parameters of an ADC as offset and gain error influence on conversion uncertainty but are not important when SNR or SINAD are calculated. Such processes as noise and quantization define values of SNR and THD and are important for both ADC types. Let us discuss these terms in detail taking into account their importance for SDADC.

According to the definitions from standard [1], "**noise (total):** any deviation between the output signal (converted to input units) and the input signal except deviations caused by linear time invariant system response (gain and phase shift), or a DC level shift". Total noise is divided into random noise and other constituents. "**Random noise:** a nondeterministic fluctuation in the output of ADC, described by its frequency spectrum and its amplitude statistical properties". Electronic [2] or thermal [3] noise of SDADC is a typical example of random noise. It usually has constant spectral density vs. frequency (white noise) and is described by the normal distribution law.

According to the opinion of the authors, other constituents can be considered as random quantities when input signal changes. At that rate, some constituents become practically random quantities at any change of input signal, while other ones become a random quantity under the condition that input signal is a random variable. Typical examples are aperture uncertainty for the former and integral linearity error for the latter variables.

According to [1], "**quantization:** a process in which the continuous range of values of an input signal is divided into nonoverlapping subranges, and to each subrange a discrete value of the output is uniquely assigned".

The quantization noise of SDADC has unique nature with regard to other ADCs. It consists of two parts. The first part is well known quantization noise produced by chosen number of bits. Industry offers many models for 24-bit SDADCs without missing codes [3, 5, 6]. Then quantum for typical full scale V_{FS} = 5 V is Q = 0.3 mV. A consumer can choose more reasonable number of bits N, of course. To a first approximation, this noise (let us use QN1 name for this process) has the uniform distribution law within limits ±0.5Q and constant spectral density from zero frequency to the half of sample frequency. Electrical noise as well as QN1 is supposed to be independent on input signal. The second part of quantization noise (let us use QN2 name for this process) is produced by the modulator of a SDADC. Just this noise is investigated in many papers concerning SDADC performance [7-8]. However, process of this noise creation is not the quantization of ADC in accordance with definition [1] given above. "Input signal is divided into nonoverlapping subranges" (two) by the comparator in sigma-delta modulator but not in the whole SDADC. This noise has strong dependence of spectral density vs. frequency; usually spectral density is proportional to f^{2n}, where n is the order of the sigma-delta modulator [7, 9]. It is theoretically possible to predict the value of QN2 for given circuit at any moment if initial conditions are known. Therefore QN2 is not random noise from this point of view. But QN2 as against QN1 changes vs. time even for constant value of input signal. Taking into account that initial value of SDADC is usually unknown or ignored, behavior of QN2 can be considered as a random process (maybe quasi-random noise is a better term). The distribution law of QN2 is not investigated in detail. According to practical recommendation of [3], standard deviation of the noise must be multiplied by 3.3 to find a maximum error. It can be derived therefore that the distribution law of QN2 is close to the normal one. The dependence of QN2 on input signal is not examined in detail also and is not reflected in specifications of produced units. The most companies give noise parameters only near zero input [3]. However, typical dependence of noise vs. input signal for the ADS1240 [5] shows that, at the ends of the input range, the average RMS of noise is two times more than that at zero input signal.

2. ERRORS OF ANALOG-TO-DIGITAL CONVERSION

Absolute static error of analog-to-digital conversion can be written through ADC parameters specified by most manufactures [3, 5, 6] as

$$\Delta = V_O + V_{IN.DIF} \cdot d_G + \Delta_Q + g_{INL} V_{FS} + \frac{\Delta V_{CM}}{CMRR} + \frac{\Delta V_{DM}}{NMRR} + \frac{\Delta V_{CC}}{PSRR} + TC_0 \cdot (T - T_0) + TC_G \cdot V_{IN.DIF} \cdot (T - T_0), \qquad (1)$$

where

V_0 – input offset which consists of systematic and random parts (see Section 3 for details).

$V_{IN.DIF}$ – average value of input differential signal. In the case of instrumentation ADC the input signal is DC voltage.

d_G – gain error (it is here in dimensionless form) which consists of systematic and random parts (see Section 3 for details).

D_Q – quantization error of ADC which is produced by QN1 (see Section 1) and is found within limits ±0.5Q = ±V_{FS} / 2^{N+1}.

g_{INL} – integral linearity error in parts of full scale V_{FS}, calculated by end point fit [1,5]. It changes smoothly vs. input signal. Dependence of capacitances on voltage in the modulator, the effects of integrator amplifier finite gain etc. are the sources of this nonlinearity [10].

ΔV_{CM} and ΔV_{CC} – deflections of common mode input voltage and power supply voltage correspondingly with regard to the values at which calibration was carried out.

ΔV_{NM} – AC input differential signal with the frequency of power line which is usually 50 or 60 Hz.

CMRR, *NMRR*, *PSRR* – common mode-, normal mode- and power supply- rejection ratios correspondingly.

TC_0 and TC_G – temperature coefficients of systematic parts for input offset and gain error correspondingly.

T and T_0 – true temperature value of ADC and the temperature, at which calibration was carried out, correspondingly.

Dynamic errors as well as static ones can be examined due to (1). In the case of instrumentation ADC, input signal is usually of low frequency, that is why the dynamic error is negligible. A sample-hold device is used for fast input signals and dynamic characteristics of ADC are not very important again. Therefore only static errors are investigated in this paper. According to (1), linear dependence of total error vs. each influence quantity is supposed excluding such quantity as input signal. Mutual action of different influence quantities is ignored in (1) excluding input signal and temperature.

Let us designate each term of (1) as Δ_i. The most positive and most negative values of systematic parts of Δ_i are designated correspondingly as $\Delta_{i.\max}$ and $\Delta_{i.\min}$. For the most cases $\Delta_{i.\max} = -\Delta_{i.\min}$. Random error is supposed to be essential only for V_0 and d_G. Maximum standard deviations for devices of given type are designated as $s[V_0]$ and $s[d_G]$. The distribution laws of Δ_i in metrology are usually supposed to be uniform [2]. If all constituents of (1) are independent, then standard deviation of error for ADC of given type can be found as

$$s[\Delta] = \sqrt{s^2[V_0] + V_{IN.DIF}^2 s^2[d_G] + \sum_{i=1}^{m} \frac{(\Delta_{i.\max} - \Delta_{i.\min})^2}{12}}, \quad (2)$$

where m – the number of error constituents; m=9 in (1) but can be increased if additional sources of error would be taken into account. The distribution law of total error is usually supposed to be normal. Then the uncertainty with typical probability P = 0.95 is $\varepsilon \approx \pm 2s[\Delta]$.

Let us discuss possibility to evaluate the uncertainty of conversion using (1) and electrical characteristics of SDADC available. As an example, specifications of three SDADC are discussed. Two devices are produced by Analog Devices (AD7714 and AD1555/AD1556), while ADS1240 is manufactured by Burr-Brown from Texas Instruments. The rated parameters of the three devices [3, 5, 6] are presented in Table 1. All the devices have nominal analog power supply voltage of +5 V, programmable gain amplifier (PGA) with gain ranging from 1 to 128, variable output data rate. The output data rate equals to the first AD7714 and AD1555/AD1556 digital filter notch frequency, while the ADS1240 first notch frequency remains constant (50 or 60 Hz typically) at various data rate values. Parameters of noise will be examined later.

Let us discuss the data given in Table 1. The authors found that specifications of given SDADCs as well as many others did not allow the evaluation of the uncertainty of conversion with chosen probability for any conditions. This statement is confirmed by following facts.

A. Some parameters of (1) are not included in data sheets. For example, V_0 and d_G are not specified for the AD7714, g_{INL} is not specified for AD1555/AD1556.

Table 1. ADC rated parameters.

Parameter	SDADC type			Units
	AD7714	ADS1240	AD1555 AD1556	
$\|V_0\|$, max	–	$37.5 \cdot 10^{-6}$	0.06	V
$\|d_G\|$, max	–	0.005	10	%
$\|\gamma_{INL}\|$, max	15	15	–	ppm
CMRR, min	100 at DC	100 at DC	93 at 200Hz	dB
NMRR at 50, 60 Hz	100, min	100, min	–	dB
PSRR	70 at dc, typical	80 at dc, min	50 at ac min	dB
$\|TC_0\|$ typical	0.4	0.1	6	mV/ °C
$\|TC_G\|$ typical	0.2	0.5	15	ppm
V_{FS}	5/PGA	5/PGA	4.5/PGA	V
PGA	1...128	1...128	1...128	-
V_{CM} range	0...5	0...5	±2.25	V
Temp. range	–40…+85	–40...+85	–55…+85	°C

B. Many parameters are specified for PGA=1 only or without indication of PGA value. This note corresponds to g_{INL}.

C. Some parameters (*PSRR* for AD7714, TC_0 and TC_G for all devices) are represented by typical values only.

This incomplete volume of information is not very important if a calibration is carried out [3]. The calibration at any temperature decreases influence of parameters V_0, d_G, TC_0 and TC_G as low as the random noise level [3]. The microprocessor based error correction technique makes it possible to decrease all systematic errors [11]. Then random noise becomes the main problem.

The maximum AD1555/AD1556 RMS noise value [6] is specified only for zero input signal ("inputs shorted together") and at a certain data rate value. Practically maximum noise RMS can be up to 5 dB (1.8 times) more than typical values. Much data for typical RMS noise values measured at zero input signal under various conditions (different data rates, gains etc.) are presented in [3]. Typical noise values are given in [5] for all input range but only for one gain and one data rate. The authors have found in [9] the minimum number of parameters which is necessary to evaluate standard deviations of random noise for combinations of gains, data rate values etc. But the dependence of the noise on input signal was not examined. This drawback is removed in the next section.

3. RANDOM ERRORS OF SIGMA-DELTA ADC

We offer the following algorithm for noise parameters definition from experimental results. The algorithm allows one to find the uncertainty of conversion within full input range using the minimum number of measured and specified parameters. From (2) the RMS of random error is found as

$$s[\Delta] = \sqrt{s^2[V_0] + V_{IN.DIF}^2[d_G]}. \qquad (3)$$

According to (3), maximum values of two parameters ($s[V_0]$ and $s[d_G]$) must be specified to calculate the maximum standard deviation of random noise for any value of input signal $V_{IN.DIF}$. To find values of mentioned parameters two measurements of ADC noise must be carried out for two values of input signal. It is very naturally to chose $V_{IN.DIF} = 0$ for the first measurement. Then the standard deviation of noise equals to $s[V_0]$. The second input signal value from the accuracy point of view must be very close to the end of the input range. However, ADC must operate yet in linear regime taking into account the maximum possible noise. Then the next parameter of (3) can be found as

$$s[d_G] = \frac{\sqrt{s_{IN.END}^2[\Delta] - s^2[V_0]}}{V_{IN.END}} \qquad (4)$$

The limits of admissible values for $s[V_0]$ and $s[d_G]$ must be written in the specifications of each instrumentation ADC. Only under this condition the uncertainty of analog-to-digital conversion can be found with given probability as was shown in Section 2. The algorithm described above is valid in general for all types of ADC. However, SDADC has important peculiarity in comparison with all others. It is a very strong dependence of noise standard deviation produced by QN2 noise (see Section 1) on input signal. Figure 1 presents an example [5].

The example shows that the measurement of noise at one input signal leads to an erroneous result. That is why we suggest the following method which is valid for all types of ADC including SDADC with significant QN2. The main point of the method is an idea that the standard deviation of noise is a random quantity. This random quantity is realized by a value at each given value of input signal. Then well known method for evaluation of mean and standard deviations can be used. For example, it is possible to take about 30 values of input signal near zero and also about 30 values near the end of the range. At each point the standard deviation is found. Then average values $s_{AV}[V_0]$ and $s_{AV.END}[\Delta]$, standard deviations of standard deviations are calculated: $s_{s.0}$ and $s_{s.END}$.

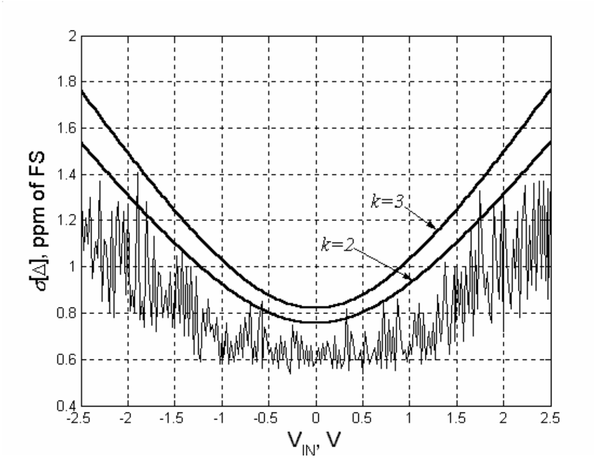

Figure 1. ADC noise vs. input signal

For uncertainty evaluation by (2) or/and standardization it is supposed that

$$s[V_0] = s_{av}[V_0] + ks_{\sigma.0} \qquad (5)$$

and

$$s[d_G] = \frac{\sqrt{\left(s_{AV.END} + ks_{\sigma.END}\right)^2 - s^2[V_0]}}{V_{IN.END}} \qquad (6)$$

The value of coefficient k depends on the importance of evaluation. The choice $k = 2$ seems to be acceptable for the most applications. The choice $k = 3$ is possible for very important cases. Both variants are shown in Fig. 1. The algorithm suggested does not require too large amount of data (choice of about 30 points near zero and about 30 points near the range end seems to be acceptable), allows us to eliminate rough errors and predict the uncertainty of conversion within full range. The algorithm described was also investigated for the SDADC simulated in the next section.

4. SIMULATION RESULTS

For the investigation of quantization noise a mathematical model of SDADC on the basis of second order sigma-delta modulator was built, the model is shown in Figure 2.

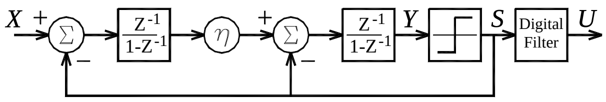

Figure 2. Model of sigma-delta ADC

The circuit consists of two parts: a sigma-delta modulator and a digital filter. The modulator was chosen of a very popular type [7]. Normalized input signal X is varied from –1 to 1. Feedback signal S from comparator has two possible levels 1 or –1 depending on the signal Y sign. A factor h can be varied. According to a recommendation given in [7], the most part of results were carried out for $h = 0.5$.

A digital filter is a 3-stage low-pass filter of moving average which transfer characteristic is

$$H(z) = \left[\frac{1 - z^{-K}}{K\left(1 - z^{-1}\right)}\right]^3, \qquad (7)$$

where K – aperture of the filter used for averaging. It is the same as oversampling term used widely [7-9]. The value of K can be set by an experimenter. Sigma-delta ADCs of many types are built on the basis of this scheme, for example [3].

All elements were supposed to be free of noise. Zero initial conditions were supposed for the filters. Then constant input signal is modeled at the input. Output signal U is registered excluding two first data samples and processed as a random quantity.

Computer simulation gave the following preliminary results:

1. Average value of U for all examined X values within linear range aspires to X when number of data is increased.
2. Standard deviation of U for all examined X values within linear range aspires to constant value (let name it as s_K) when number of data is increased.
3. The distribution law of U for all examined X values within linear range is close to normal one if $X\text{-}3s_K < U < X\text{+}3s_K$.
4. Standard deviation s_K changes very sharp (several times) even for very small change of X (see Figure 3 and Figure 4).
5. Standard deviation s_K can be considered as a random quantity with mean $M[s_K]$ and standard deviation $\sigma[s_K]$ if X is considered as a random quantity. The distribution law of s_K is close to the normal one (see Figure 5) if at least

$$M[s_K] - 2\sigma[s_K] < s_K < M[s_K] + 2\sigma[s_K].$$

6. Quantity $M[s_K]$ is an even function of X and increases vs. X (see Figure 3). Maximum values of $M[s_K]$ correspond to the ends of linear range.
7. Noise $M[s_K]$ can be predicted by (9) from [8] within the range $|X| \le \beta$ (see Figure 3) with uncertainty of about $\pm\sigma[s_K]$. The same result was found by the authors before for the noise of AD7714 at zero input [9].
8. Within the whole linear range (between two maximums at Figure 3) the uncertainty of conversion can be evaluated by (3).

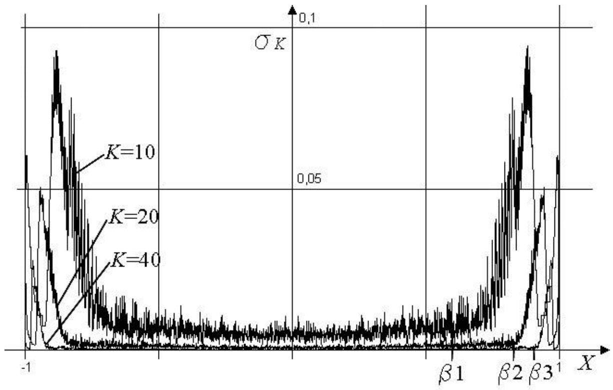

Figure 3. Simulated sigma-delta ADC noise RMS vs. input signal.

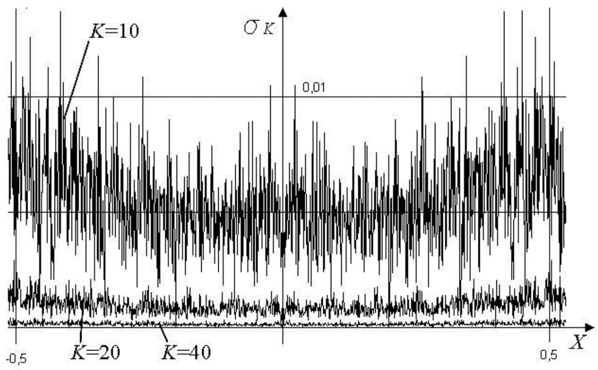

Figure 4. Simulated sigma-delta ADC noise RMS vs. input signal for |X| < 0.5.

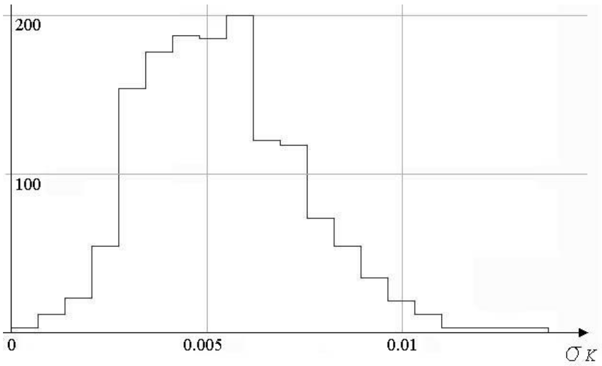

Figure 5. Distribution of s_K vs. X.

5. CONCLUSIONS

Random noise of modern instrumentation sigma-delta analog-to-digital converters dominates when evaluating the conversion uncertainty. This random noise consists of electronic and quantisation (QN2) parts. The equations derived by the authors before [8], and simulation (section 4 of the given paper) allow the prediction of the uncertainty of SDADC with second-order modulator such as AD7714 [9] at zero input signal at least. Two parameters ($s[V_0]$ and $s[d_G]$) in (3) must be measured or/and specified in order to evaluate the uncertainty of conversion with given probability within the whole linear range of input signal.

REFERENCES

[1] IEEE Standard 1241 for terminology and Test Methods for Analog-to-Digital Converters, draft, May, 1999.

[2] V.I. Didenko, V.P. Fedotov: "The evaluation of measurement uncertainty". Journal of Electrical Engineering, vol. 44, No. 2, 1993, pp. 33-37.

[3] Signal Conditioning AD7714, *Analog Devices,* U.S.A., Data Sheet, 1997.

[4] STANDARD 8.009-84. Specified metrological characteristics of measuring instruments (in Russian).

[5] 24-Bit Analog-to-Digital converter ADS1240, *Burr-Brown products from Texas Instruments*, June 2001.

[6] 24-Bit Σ-Δ ADC with Low-Noise PGA. AD 1555/AD1556, *Analog Devices,* U.S.A., Data Sheet, 2000.

[7] P. Benabes, P. Alderebt, R. Kielbasa, Analog-to-digital sigma-delta converters modeling for simulation and synthesis, *Proceedings of International Workshop on ADC Modelling and Testing*, Bordeaux, France, September 9-10, 1999, pp.3 -14.

[8] V.I. Didenko, I. Kale, A.L. Movchan, Y.S. Solodov. "Metrological Modelling of σ–δ ADC. *Proc. of 6th Workshop on ADC Modelling and Testing* , Lisbon, Portugal, September 13 -14, 2001, pp. 14-18.

[9] IEEE V. Didenko, A. Movchan, “Minimization of number of metrological parameters for data acquisition systems”, *IEEE Transactions on Instrumentation and Measurement*, vol. 51, No. 1, Feb. 2002, pp. 88-91.

[10] J.M[a] Gomes Cama, S.A. Bota, E. Montana, J. Smitier, "Modeling, Design and test of a mosfet-only second order delta-sigma modulator", *Proceedings of International Workshop on ADC Modelling and Testing*, Bordeaux, France, September 9-10, 1999, pp.15 -20.

[11] V. Didenko, A. Minin, A. Movchan, “Polinomial and piece-wise linear approximation of smart transducer errors”, *Measurement*, vol. 31, No. 1, Jan. 2002, pp. 61-69.