AMPLIFIER IMPERFECTION EFFECTS IN SWITCHED-CAPACITOR RESONATORS

*Mustafa Keskin**, *Un-Ku Moon, and Gabor C. Temes*

Department of Electrical & Computer Engineering, Oregon State University, USA
*Qualcomm Inc., USA, mkeskin@qualcomm.com, Fax: (858) 845-6832

1. ABSTRACT

A switched-capacitor bandpass analog-to-digital-converter is one of the circuit blocks used in wireless communication systems to digitize the received analog signal at certain center frequency (f_c). This particular converter is used especially for digital FM or AM radio applications and most of the portable communication devices such as cellular-phones. The main block of this converter is the resonator, which resonates at this f_c center frequency. Two of the main design criteria of the resonators are; to have very high quality resonance peak and to locate the center frequency accurately without any shift. However, because of the circuit imperfections, the resonance peak gain and the center frequency have degraded in the previous well-known architectures, seriously.

2. INTRODUCTION

The main block of bandpass converters is a switched-capacitor (SC) resonator. It is very important to design less-sensitive SC resonators to amplifier imperfections in order to push analog-to-digital conversion towards the antenna. In order to build accurate systems, the modeling of resonators and bandpass modulators should be performed both in circuit and also in system level [1]-[2].

There exist many resonator circuits to implement SC bandpass $\Delta\Sigma$ modulators and filters for high-frequency communication applications. In this paper, two-integrator loop [3], two-delay loop [4]-[5], pseudo-N-path [6], integrating-two-path [7]-[8], and direct-charge-transfer pseudo-N-path (DCT-PNP) [9]-[10] are discussed.

The typical resonator transfer function with unit delay from input to output is given by $H(z) = z^{-1}/(1+z^{-2})$. This leads to the time-domain relation $v_o(n) = v_{in}(n-1) - v_o(n-2)$, which involves a delay by $2T$ and the inversion of $v_o(n)$.

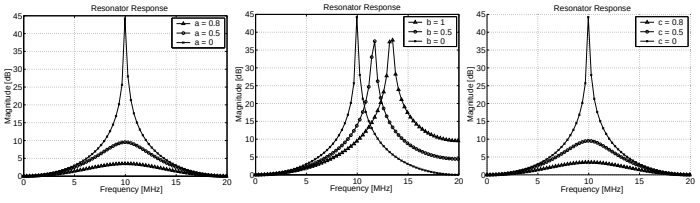

Figure 1: The simulation results with different values of error term (1) 'a' (2) 'b' (3) 'c'

There are two important criteria for high-accuracy charge transfer using an opamp. These are settling-time of a particular stage and DC-gain (A_{dc}) of an opamp. Ideally, A_{dc} and the unity-gain-bandwidth (f_u) should be infinite to have perfect charge-transfer.

The ideal transfer function given is in practice degraded because of the nonidealities of the circuit, specifically because of the nonideal opamp into

$$H(z) = \frac{z^{-1}}{(1-a) + bz^{-1} + (1-c)z^{-2}} \quad (1)$$

The peak gain is reduced due to the error terms a and c, which affect the quality factor Q of the resonance. The shift in center frequency is introduced by the error term b. The effects of these terms are shown in Fig. 1 separately.

3. THE TWO-INTEGRATOR LOOP RESONATOR

The two-integrator loop (TIL) resonator which can be implemented with two cascaded half-delay integrator in a positive feedback loop is shown in Fig. 2 in single-ended configuration for illustration purpose only.

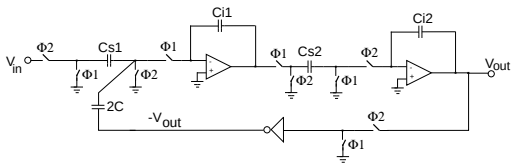

Figure 2: Single-ended TIL resonator

Since the $H(z)$ of this resonator can be obtained from two integrators, the effects of nonidealities on such integrator will be analyzed next.

3.1. Analog Imperfections in switched-capacitor Integrator

In the presence of finite A_{dc}, the half-delay SC integrator will have the transfer function $H_1(z) = (m_1 z^{-1/2})/(1-p_1 z^{-1})$ where m_1 is the actual integrator gain and p_1 is the shifted pole. They are given by

$$m_1 = \frac{(\frac{C_{s1}}{C_{i1}})}{1+\frac{1}{A_{dc}}(1+(\frac{C_{s1}}{C_{i1}}))} \; and \; p_1 = \frac{1+\frac{1}{A_{dc}}}{1+\frac{1}{A_{dc}}(1+\frac{C_{s1}}{C_{i1}})} \quad (2)$$

where C_{s1} and C_{i1} are the sampling and integrating capacitors of the integrator, respectively.
The finite f_u introduces a gain error in an ordinary integrator $H(z) = (C_{s1}(1-g_1)z^{-1/2})/(C_{i1}\cdot(1-z^{-1}))$ where the gain error term is $g_1 = e^{-T/\tau}$, and τ is the settling time constant.

3.2. The Finite Opamp-Gain and -Bandwidth Effects

The analysis can be easily performed by utilizing signal-flow-graph method. Fig. 3 shows the error terms caused by the effects for TIL resonators, mentioned earlier. The effect of the finite gain will be

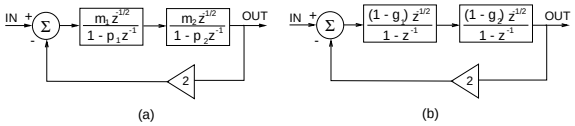

Figure 3: TIL resonator with error terms caused by (a) finite opamp-gain and (b) finite opamp-bandwidth

severe in TIL. It will not only introduce gain error but also shift the center frequency as shown below:

$$H(z) = \frac{(m_1 m_2)z^{-1}}{1 + (2m_1 m_2 - p_1 - p_2)z^{-1} + (p_1 p_2)z^{-2}} \tag{3}$$

where m_1 and p_1 refer to the first integrator and m_2 and p_2 refer to the second integrator. The effects of the finite bandwidth is shown in the Eq. 4. There is frequency shift in this case, too.

$$H(z) = \frac{G(1 + g_1 g_2 - g_1 - g_2)z^{-1}}{1 - (2g_1 + 2g_2 - 2g_1 g_2)z^{-1} + z^{-2}} \tag{4}$$

The limited bandwidth not only shifts the center frequency and introduces out-of-band noise into the signal band, but also sets an upper limit on the sampling frequency, because g increases with sampling frequency and severely affects the shift.

4. THE TWO-DELAY-LOOP RESONATOR

The two-delay-loop (TDL) resonator is preferred over TIL resonator because of its relaxed settling time requirements [4]-[5]. The TDL resonator architecture is shown in Fig. 4.

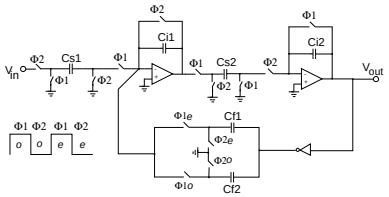

Figure 4: Single-ended TDL resonator

In this circuit, a half-clock-cycle ($T/2$) delay is provided by the sample-and-hold (SH) circuit. Hence, a full (T) delay is introduced by the two cascaded SHs in the forward path. The feedback capacitors C_{f1} and C_{f2} keep the charge for T, consecutively, and hence add a T delay in the feedback path to insure the resonator operation.

4.1. Analog Imperfections in a Delay Cell

In the presence of finite gain, the half-delay SC cell will have the transfer function as $H_1(z) = m_1 z^{-1/2}$ where m_1 is the actual gain of the half-delay cell. It is the same as that (m_1) of a SC integrator as shown in Eq. (2). There is no pole error in a delay cell.

The finite bandwidth of an operational amplifier introduces a gain error in a delay cell: $H(z) = C_s(1 - g_1)z^{-1/2}/C_i$.

4.2. The Finite Opamp-Gain and -Bandwidth Effects

The error terms of TDL resonators are shown in Fig. 5. The finite gain only introduces gain error at the resonance peak in TDL resonator. This transfer function with error term is shown below:

$$H(z) = \frac{(m_1 m_2)z^{-1}}{1 + (m_1 m_2)z^{-2}} \tag{5}$$

where m_1 refer to the first delay cell and m_2 refer to the second delay cell.

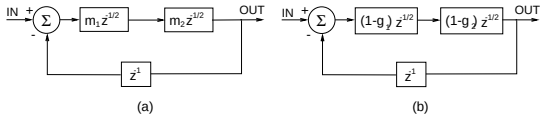

Figure 5: TDL resonator with error terms caused by (a) finite opamp-gain and (b) -bandwidth

There is no center frequency shift in TDL resonator. The coefficient of z^{-2} term is equal to $(m_1 \cdot m_2)$ instead of "1" as for the ideal case. Therefore, a gain drop will be introduced.

The effect of finite bandwidth can be calculated using Fig. 5. The transfer function becomes to

$$H(z) = \frac{(1 + g_1 g_2 - g_1 - g_2)z^{-1}}{1 + (1 + g_1 g_2 - g_1 - g_2)z^{-2}} \tag{6}$$

The limited bandwidth causes similar results to those of the limited gain. The integration of the error terms are eliminated since, unlike in the TIL resonator, there is no integration at any time either at the forward or at the feedback path. On the other hand, this configuration thus requires opamps with high DC gain since opamps are used as buffers.

5. PSEUDO-N-PATH RESONATOR USING OPAMP CHARGE TRANSFERS

The ideal transfer function of the resonator considered is $H(z) = z^{-1/2}/(1 + z^{-2})$. This translates into the discrete-time input-output relation $v_o(n) = v_{in}(n - 1/2) - v_o(n - 2)$. Hence, in the PNP resonator shown in Fig. 6(a) the new output contains the input as well as the output inverted and delayed by two clock cycle. In this structure, the differential output voltage is provided by two capacitors, say $C1p$ and $C1n$ during clock phase 'a'. This output voltage is stored on the same capacitors for two clock periods. Then $C1p$ and $C1n$ are interchanged, and they transfer their charges to the new feedback capacitors, $C3p$ and $C3n$ during clock phase 'c'. This operation provides the delayed output. Unfortunately, due to imperfect charge transfer, it also integrates some error signal charges with a delay T. Hence there is a z^{-1} term generated in the denominator of $H(z)$.

5.1. Finite Opamp-Gain and -Bandwidth Effects

The analysis is first performed in the time domain. Fig. 6(b) shows the circuit during phase 'a'. For a finite opamp gain A_{dc}, charge conservation gives

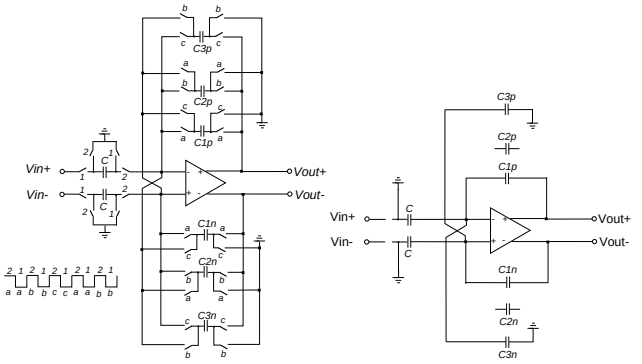

Figure 6: Pseudo-N-path SC circuit with opamp-aided charge transfer (a) whole (b) during phase 'a'

$$C1\left(V_o(n)+\frac{V_o(n)}{A_{dc}}\right)=C3\left(-V_o(n-2)-\frac{V_o(n-2)}{A_{dc}}\right)$$
$$-C1\frac{V_o(n-1)}{A_{dc}}-C3\frac{V_o(n)}{A_{dc}}+C\left(V_{in}(n-1/2)-\frac{V_o(n)}{A_{dc}}\right) \quad (7)$$

Ideally, $C1$ is equal to $C2$ and $C3$. For simplicity, they will be denoted by Ci. Then, from Eq. (7),

$$Ci\cdot V_o(z)\left(1+\frac{1}{A_{dc}}+\frac{z^{-1}}{A_{dc}}+z^{-2}+\frac{z^{-2}}{A_{dc}}+\frac{1}{A_{dc}}+\frac{C/C_i}{A_{dc}}\right)=C\cdot V_{in}(z)\cdot z^{-1/2} \quad (8)$$

and

$$H(z)=\frac{V_o(z)}{V_{in}(z)}=\frac{C/C_i\cdot A_{dc}\cdot m\cdot z^{-1/2}}{1+m\cdot z^{-1}+(1+A_{dc})\cdot m\cdot z^{-2}} \quad (9)$$

where $m=(1)/(2+A_{dc}+C/C_i)$.

As seen from Eq. (9), both center frequency shift and resonant gain loss occur in the opamp-aided charge-transfer PNP resonator.

The finite bandwidth effects of the opamp can also be calculated using time-domain equations. Incomplete linear settling can be modelled by multiplying the input charges by $(1-g)$, where the gain error term is $g=e^{-T/\tau}$, and τ is the settling time constant. Also, there is some charge kept by the $C1$ capacitors from two clock cycles before due to incomplete settling. The time-domain equation is

$$C1\cdot V_o(n)=(1-g)\left(C\cdot V_{in}(n-1/2)-C3\cdot V_o(n-2)\right)+g\cdot C1\cdot V_o(n-1). \quad (10)$$

When $C3$ and $C1$ are replaced by C_i,

$$H(z)=\frac{C}{C_i}\frac{(1-g)\cdot z^{-1/2}}{1-g\cdot z^{-1}+(C/C_i)\cdot(1-g)\cdot z^{-2}} \quad (11)$$

results. Hence, both gain loss and center frequency shift occur in the PNP resonator, when the opamp used has finite bandwith.

6. PSEUDO-N-PATH RESONATOR WITH DIRECT CHARGE TRANSFER

The DCT-PNP circuit [9]-[10] shown in Fig. 7. In this structure, $C1p$ and $C1n$ hold the differential output voltage $v_{out}(n)$ during clock phase 'c'. At the same time, $C3p$ and $C3n$ are connected between the output and the input of the resonator, and hence they are charged to $v_{out}(n)-v_{in}(n)$. Two clock periods later, the $C3$ capacitors are connected as feedback capacitors during clock phase 'b', thus providing $v_{out}(n+2)$.

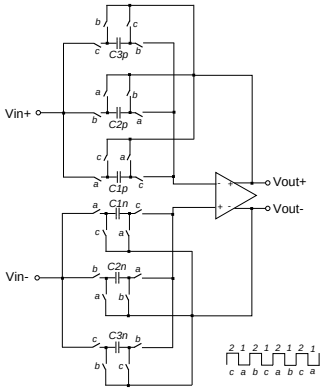

Figure 7: Direct-charge-transfer resonator

In this DCT-PNP resonator, no charge transfer through the virtual ground is needed to realize the resonator transfer function. Therefore, the finite bandwidth does not affect the accuracy of the resonating frequency. The finite gain introduces only a reduction of the "Q" of the resonator as shown by $H(z)=z^{-2}/(1+(1-d)\cdot z^{-2})$ where d is the resonant gain error given by $d=1/A_{dc}$.

This operation does not integrate error charges and hence there is no center frequency shift. The noncritical error term d merely reduces the quality factor Q of the resonance. Another advantage of the DCT-PNP circuit is that it is not affected by capacitor mismatches since the same capacitor samples the input and substracts it from the previous output (by charge sharing between capacitors) to provide the delayed output. There are only small second-order mismatch effects due to the charge on the top-plate parasitic capacitors.

7. THE INTEGRATING-TWO-PATH RESONATOR

In the I2P structure [7]-[8] shown in Fig. 8, the differential voltage which occured two clock period earlier is stored in the C_1 capacitors during odd clock phases. Then this pair is interchanged in the next clock phase to realize the resonator input-output relation. The same operation is done by the other pair C_2 in even clock phases. This operation prevents the introduction of odd-order terms into the denominator of H(z), since no error signal delayed by only one clock period can occur.

7.1. The Finite Opamp-Gain and -Bandwidth Effects

The errors in I2P architecure are the same errors occuring in an integrator. I2P resonator can be thought as a double-delayed integrator in its feedback. Therefore simple analysis shows the same error terms as those of the lossless-discrete integrator. The transfer function is $H(z)=m\cdot z^{-1/2}/(1+pz^{-2})$ where m is the actual resonator gain and p is the resonance gain error. They are the same as those given in Eq. (2). The finite bandwidth of

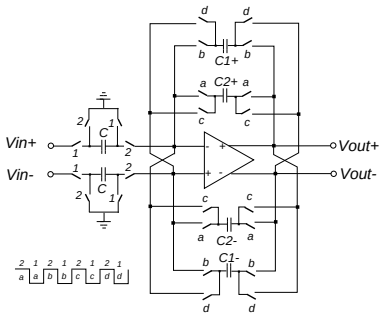

Figure 8: Integrating-2-path resonator

an operational amplifier introduces a gain error in I2P resonator; $H(z) = (C_s(1-g)z^{-1/2})/(C_i \cdot (1+z^{-2}))$ where the gain error term is $g = e^{-T/\tau}$, and τ is the settling time constant.

8. SIMULATION RESULTS:

SWITCAP simulations were used to compare the conventional PNP, DCT-P2P, I2P, TDL, and TIL structures. In Fig. 9(a), the peak resonances of five resonators are obtained at 10 MHz for the (nearly) ideal case where the unity-gain-bandwidth ($f_u = \infty$) and the gain (A_{dc} = 120 dB) of the opamp used do not limit the performance. In Fig. 9(b), the effects of the finite gain limitation of the opamp are shown, for $f_u = \infty$ and A_{dc} = 60 dB. DCT-P2P and I2P have some gain loss, but they are better than the others. The PNP circuit has both gain loss and frequency shift. TIL is the worst case with serious drop and shift. The TDL has severe gain loss as well. In Fig. 9(c), the effects of the finite bandwidth limitation of the opamp are shown, for f_u = 100 MHz and A_{dc} = 120 dB. DCT-P2P has no gain loss or any shift in the center frequency, while the PNP and the TDL have almost 40 dB gain loss. At the same time, the I2P has just 8 dB gain loss. TIL is again the worst case.

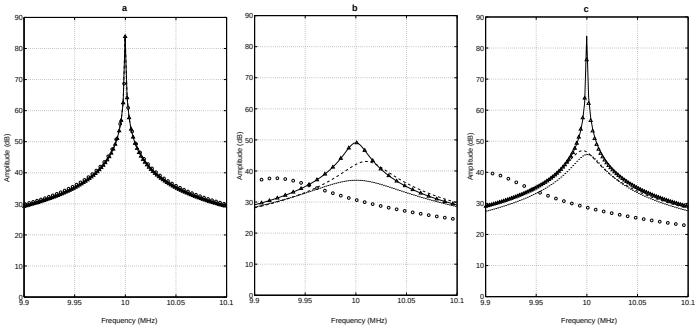

Figure 9: Simulation Results from SWITCAP for I2P (triangles), PNP (dashes), TDL (dots), TIL (circles), DCT-P2P (line) for (b) A_{dc} = 50 dB and $f_u = \infty$ (c) A_{dc} = 120 dB and f_u = 80 MHz

9. CONCLUSIONS

There are several drawbacks of the existing resonator circuits. First of all, both TDL and TIL resonators need two opamps, since each integrator or delay-cell requires one opamp. Thus, the nonidealities discussed before affect to the transfer functions of the SC resonators twice as much as that of the basic SC integrator or delay cell. Additionally, these error terms are integrated over time and mixed each other. As seen from both the simulation results and analysis, these architectures cannot alleviate the imperfections (errors) originating from the basic blocks directly. PNP resonator uses only one opamp, on the other hand it also introduces the integration of the error terms.

Hence, there is strong need for architectures such as I2P and DCT-PNP, which allow very relaxed requirements for the circuit components, and hence provide high quality and robust products for future communication devices.

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