Band Pass Sigma Delta Modulator with Tuneable Center Frequency

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Abstract – In this paper a new structure for a band-pass sigma delta modulator is proposed. The center frequency of the modulator can be tuned in the range between DC to half the sampling frequency by means of only one parameter. The modulator stability and the nearly constant resolution for the whole frequency range are demonstrated.

Keywords – Sigma delta modulator, band pass modulator, tuneable filter.

I. Introduction

The data conversion technique based on the sigma delta modulation is used for all the applications where medium and high resolutions are required. The structure of the sigma delta converter is subdivided in two sections: analog and digital parts. The first part (*modulator*) works on the analog input signal to convert it in a digital sequence of "short words" at high frequency, called *bitstream*. The second part (*decimation filter*) converts the bitstream in a sequence of "long words" at the Nyquist frequency. Combining the oversampling and the noise shaping techniques the sigma delta modulation modifies the quantization noise inside the signal bandwidth to improve the signal-to-noise ratio (*SNR*). The first technique reduces the amount of the quantization noise and the second moves it outside of the signal bandwidth. Band pass sigma delta converters (*BPSDC*) are widely used in systems where narrow band signals are present such as radio-frequency (*RF*) communication systems, spectrum analyzer or special-purpose instrumentation for narrow band sources [1] - [3]. For example in data conversion at intermediate frequency (*IF*) the BPSDC allows to move IF filtering from the analog to the digital domain where design, testing and filter parameter setting are simpler. If the center frequency of the BPSDC is tuneable the modulator performance can be optimized by adjusting the transfer function to prevent the instability of high order filter used in RF/IF systems. One solution has been proposed in [4] where the tuneable cell has been obtained by changing the transconductance of the operational amplifier inside the resonator loop, but the used continuous time technique is not compatible with a low-cost CMOS technology. In this paper a novel one parameter only BPSDC tuneable modulator is proposed. It is based on a tuneable resonator, which can be implemented with switched capacitor technique in standard CMOS process. Moreover, being the control parameter value determined by a capacitor ratio, the circuit is quite robust against process parameter variations.

II. Algorithm

The design of a 2nth order band pass modulator requires a nth order low pass modulator prototype and the low pass to band pass transformation. The chosen structure to implement the prototype is called Chain of Integrators with distributed FeedBack (*CIFB*) [3]. In this simple structure the filter transformation can be applied directly to the delay block, z^{-1} by replacing the integrator block with a *resonator*. The general transformation from low pass to band pass is [5]:

$$z^{-1} \rightarrow -\frac{z^{-2} - \frac{2\alpha k}{k+1} z^{-1} + \frac{k-1}{k+1}}{\frac{k-1}{k+1} z^{-2} - \frac{2\alpha k}{k+1} z^{-1} + 1} \quad (1)$$

where the parameters α and k are defined below:

$$\alpha = \frac{\cos((\omega_2 + \omega_1)/2)}{\cos((\omega_2 - \omega_1)/2)} = \frac{\cos(\omega_c)}{\cos(B/2)}$$
$$k = \frac{\tan(\theta_p/2)}{\tan((\omega_2 - \omega_1)/2)} = \frac{\tan(\theta_p/2)}{\tan(B/2)} \quad (2)$$

and the frequencies ω_2 and ω_1 are the desired upper and lower cut off frequencies, $\omega_c=(\omega_2+\omega_1)/2$ is the center frequency and $B=\omega_2-\omega_1$ is the bandwidth of the band pass filter; θ_p is the cut off frequency of the low pass prototype filter. The values of α are in the range [-1,1] and the center band, ω_c, is limited between half sampling frequency, $f_s/2$, to *DC*. The range of k is between 0 to ∞ and the band pass bandwidth range is $[\pi, 0]$. To obtain the band pass tuneable modulator (*BPT*) from the low pass one, the transformation (1) has to be used. To preserve the bandwidth from low pass to band pass filter the k parameter has to be equal to 1 (from (2): $k=1 \rightarrow B=\theta_p$) so the transformation (1) becomes:

$$z^{-1} \rightarrow \frac{z^{-2} - \alpha z^{-1}}{\alpha z^{-1} - 1} \quad \text{with} \qquad \alpha = \frac{\cos(\omega_c)}{\cos(B/2)} \quad (3)$$

The transformation (3) allows the tuning of the center frequency ω_c of the band pass filter from 0 to half the sampling frequency (*fs/2)* by means of the α parameter. Note that $\alpha=0$ corresponds to the classical BPSDC transformation.

III. Resonator

Applying the transformation (3) to the delay block z^{-1} the integrator of the low pass prototype modulator becomes resonator. This is shown below:

$$S_{LP}(z) = \frac{B(z)}{A(z)} = \frac{z^{-1}}{1 - z^{-1}} \rightarrow S_{BPT}(z) = \frac{B(z)}{A(z)} = \frac{\alpha z^{-1} - z^{-2}}{1 - 2\alpha z^{-1} + z^{-2}}$$

Figure 1 shows a possible realization of such a resonator. The BPT modulator is then derived by replacing each integrator in the classical CIFB structure with a resonator. By designing the resonator in order to have the filter's stop band coinciding with the signal spectrum, the quantization noise spectrum is shaped so that most of the noise occurs at frequencies outside the signal spectrum. Moreover, the center band of the notch filter can be adjusted by the parameter α.

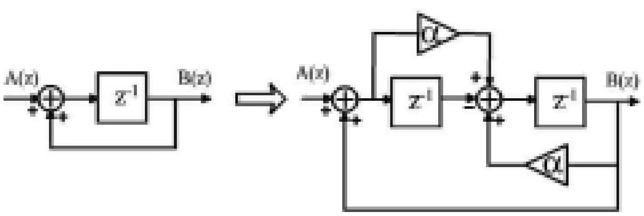

Figure 1 – Possible realization of the *resonator*.

IV. AC Linear Analysis

Modelling the quantization error as input independent additive white noise [3], the system becomes linear and AC analysis can be used to describe the behavior of the modulator. The noise and signal transfer functions (*NTF* and *STF*, respectively) for the CIFB structure of the n^{th} order modulator result:

$$NTF(z) = \frac{1}{1 + a_1 S(z)^n + a_2 S(z)^{n-1} + \ldots + a_{n-1} S(z)^2 + a_n S(z)^1}$$

$$STF(z) = \frac{S(z)^n}{1 + a_1 S(z)^n + a_2 S(z)^{n-1} + \ldots + a_{n-1} S(z)^2 + a_n S(z)^1}$$

The transfer functions of the low pass or band pass tuneable modulator can be obtained replacing *S(z)* by $S_{LP}(z)$ or $S_{BPT}(z)$ respectively.

Figure 2 shows the NTF of a fourth order BPT modulator for three different values (-*0.9*, *0* and *0.9*) of the α parameter. The normalized frequencies of the corresponding notch filters are *0.4282*, *0.25*, and *0.0718*, respectively.

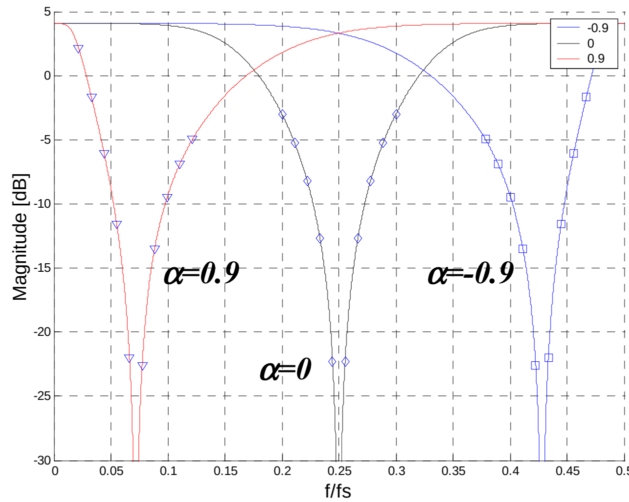

Figure 2 – *NTF* of a linear loop filter

V. Notch Filter

In the linear system the NTF notch filter rejects any signal inside a bandwidth equal to the cut off frequency of the prototype low pass filter (θ_p) and centered at ω_c. The center frequency of the notch filter band can be obtained directly from the definition of the parameter α:

$$\omega_c = \arccos(\alpha \cos(\theta_p/2)) \quad (4)$$

The minimum of the *NTF* notch filter is situated according to the zeros, which are n order complex conjugated pairs due to the $[1-2\alpha\, z^{-1} + z^{-2}]$ term in $S_{BPT}(z)$.

The mathematical relation between the parameter α and the notch frequency is:

$$f_{notch} = \frac{f_s}{2\pi} \arccos(\alpha) \quad (5)$$

Two frequencies, f_c and f_{notch}, are equal only for α=0 because the shape of NTF is symmetric around the notch; for $\alpha \neq 0$ the reject band is asymmetric and this asymmetry increases for values of $|\alpha|$ close to 1.

Figure 3 shows the NTF of the linear modulator, the limit of the bandwidth where the noise is rejected ($2\pi f_p = \theta_p$), the center frequency of the notch band (f_c), and the frequency of the minimum of the NTF (f_{notch}) for $\alpha=0.9$.

When $\alpha \neq 0$ the difference between f_{notch} and f_c increases until maximum value is equal to $|f_c - f_{notch}| = f_p/2$, for $|\alpha|=1$. When α moves the reject band towards to *DC* or $f_s/2$ the asymmetry of this band imposes some conditions on the bandwidth of input signal, with center frequency equal to the minimum of the notch filter f_{notch}, to avoid that some frequencies of the input signal could be outside the reject band of system.

A relationship between the properties of the input signal (bandwidth B_{sign} and center frequency f_{notch}) and the properties of the notch filter (bandwidth f_p and the center frequency f_c) can be obtained from figure 3: for $\alpha>0$ the half bandwidth of the input signal must be smaller than the distance between f_{notch} and $(f_c - f_p/2)$.

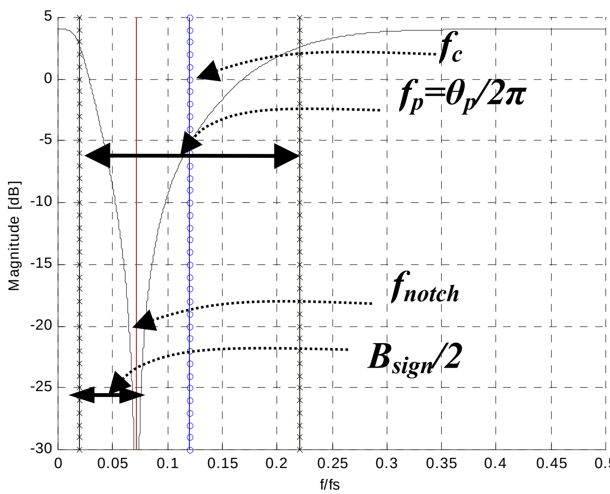

Figure 3 – Notch filter for $\alpha=0.9$, centre frequency $f_c=0.1202 \cdot f_s$ ("o"), bandwidth $f_p=0.2 \cdot f_s$ ("x") and notch frequency $f_{notch}=0.0718 \cdot f_s$.

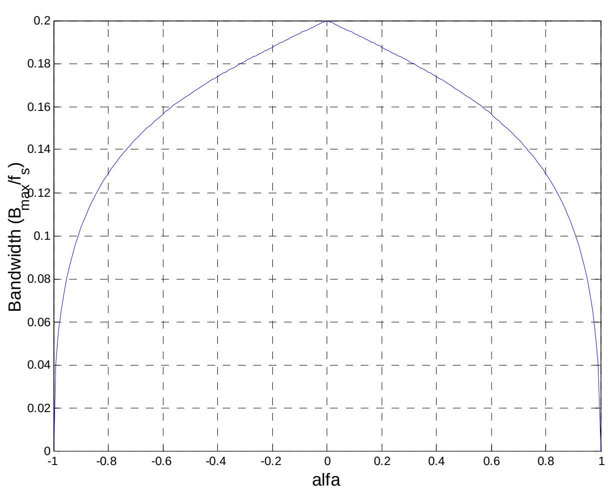

Figure 4 – Maximum Bandwidth of the input signal for $\alpha \in [-1, 1]$.

The signal bandwidth bounds are:

$$\begin{aligned} B_{sign}/2 &\leq f_{notch} - \left(f_c - f_p/2\right) \qquad \alpha \geq 0 \\ B_{sign}/2 &\leq \left(f_c + f_p/2\right) - f_{notch} \qquad \alpha < 0 \end{aligned} \quad (6)$$

For the limit cases when $\alpha \approx \pm 1$ the bandwidth of the input signal moves towards to zero. Under these limits the maximum bandwidth, B_{sign}, of the input signal with center frequency f_{notch} is related to the parameter α by (4), (5) and (6). Figure 4 shows the maximum bandwidth (B_{MAX}) needed in order to avoid negative frequencies or frequencies over Nyquist frequency. In these graphics the bandwidth of the input signal is normalized to the sampling frequency f_s. Note that the resolution of the modulator is bound by the bandwidth of the input signal.

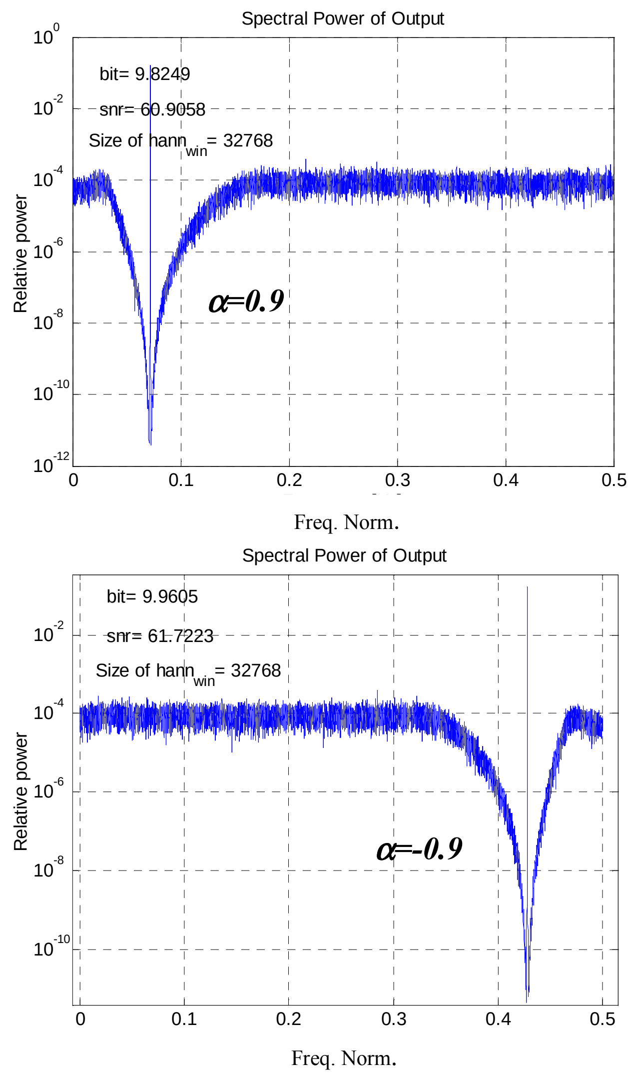

Figure 5 – Spectral power of output for values of α equal to ± 0.9.

VI. MATLAB SIMULATION

When hard-quantizer is inserted in the loop filter, previous linear model is not valid any longer. Matlab model can be used to study the behavior and to verify performance of the BPT modulator by computer simulations. Figure 5 shows the output spectrum of the fourth order BPT modulator for different values of parameter α when a sinusoidal signal of frequency f_{notch} is applied to the input. The power spectrum is obtained by applying a “Hanning” windowed FFT on 32768 output samples. Note the asymmetry of the band pass shape like shown in figure 2 from the linear analysis. The frequencies of the power spectrum are normalized to the sampling frequency. The resulting SNR for a normalized bandwidth of 0.002 is always greater than 60 dB (more than 9 bit resolution) for all the frequency range from *DC* to $f_s/2$.

VII. CONCLUSIONS

In this paper a band pass sigma delta modulator tuneable by means of one parameter only is presented. The relationship between such a parameter and the minimum of the NTF notch filter of the band pass is derived. Computer simulations for the non-linear model validate the theoretical approach.

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