A Formulation for the Synthesis of Robust Digital Correction Filters in Cascaded Sigma-Delta Converters

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Abstract – Cascaded sigma-delta converters relax the requirements on the oversampling ratio for a given resolution, but their performance is sensitive to imperfections in the analog components. Previous work has focussed on adaptive calibration schemes to address this problem. In contrast, the main contribution of the present work is the development of a framework for the synthesis of robust digital correction filters that provide a guaranteed level of performance for the worst case uncertainty in parameter values. The validity of the approach is established by successfully reproducing well-known results.

Keywords – Cascaded sigma-delta converters, robust control, software radio, structured singular value.

I. INTRODUCTION

Analog-to-digital converters (ADC) based on sigma-delta modulators are seen to be promising candidates for digitization in wide bandwidth digital communication devices such as software radios [1,2]. The high oversampling ratios (OSR) required by sigma-delta converters place limitations on the input bandwidth, and therefore, the standard architectures must be modified if they are to be used in high-speed applications. One popular approach to relaxing the requirements on the OSR is to pipeline several low-order sigma-delta stages [3]. The advantage of such cascaded architectures is that they provide effectively high-order noise shaping at relatively low OSR, whilst avoiding the conditional stability restrictions normally associated with high-order single loop topologies.

A drawback of cascaded sigma-delta converters is that they are very sensitive to imperfections in the analog components [3,4]. Typical sources of error are mismatch in capacitor values and finite amplifier gain, which translate to pole and zero errors in the noise shaping filters. Digital correction for these effects is possible, and has been demonstrated by Gabor and Temes [4]. In their work, Gabor and Temes employed an adaptive strategy to account for imprecision in the analog components. The possible disadvantages of this method depend on whether on-line or off-line calibration is used. There is some loss in the dynamic range in the on-line reference-based case, while in off-line calibration the normal operation of the converter must be disrupted. In both cases, there is an increase in the complexity of the digital circuitry.

This paper presents an approach that allows the correction filter to be fixed at the design stage and thus does not suffer the above limitations associated with the adaptive scheme. The new strategy draws on established ideas in robust control to enable filters to be designed that provide a guaranteed level of robust performance in the face of worst-case variations in the system parameters. The work concentrates mainly on developing the framework and provides preliminary results for illustration. The paper is organised as follows. First, the synthesis framework is developed following the briefest review of relevant results from robust control. In the next section, some numerical results are presented to provide some validity to the approach and an example of how the new formulation may be utilised. The paper ends with conclusions and a short discussion of future directions.

All of the results and simulations in this paper were generated using the MATLAB "μ-Analysis and Synthesis Toolbox" [5].

II. DEVELOPMENT OF FRAMEWORK

A. Theoretical Background

In this section, a brief review is given of the main ideas and results from modern control that is relevant to the present study. Further details can be found in the authoritative works on robust control such as [6] and [7].

Robust performance analysis and synthesis typically begins with converting the problem in-hand into the general control configuration of figure 1. The quantity P models the open-loop plant and is an appropriately partitioned matrix of transfer functions relating the various inputs to the outputs. The vector of control inputs and measurements are given by u and v respectively; w represents the exogenous inputs (such as disturbance and reference command signals); the error signals to be minimised in some sense are given by z. The uncertainty elements are "pulled out" into block-diagonal form, represented by $\Delta = \text{diag}\{\Delta_i\}$ in figure 1. Each of the Δ_i represents a particular source of uncertainty. Finally, K is the transfer function of the controller to be designed or analysed.

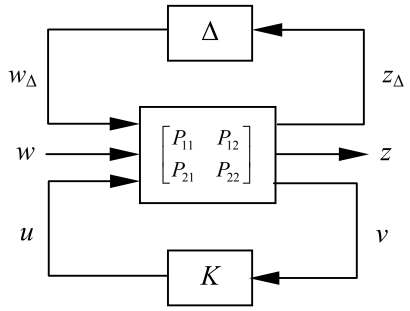

Fig. 1. General control configuration.

Typically the control problem is tackled by considering the size in some sense of the closed-loop transfer function between the error signals, *z*, and the exogenous inputs, *w*. Design for robustness leads to minimization of the H-infinity norm, which is written:

$$\begin{aligned} \|F\|_\infty &= \|F_u(N,\Delta)\|_\infty \\ N &= F_l(P,K) \end{aligned} \tag{1}$$

where F_u and F_l are upper and lower linear fractional transformations (LFT) respectively. The lower LFT between the partitioned matrix *P* and the controller *K* is defined as follows (the upper LFT is defined in an analogous manner):

$$F_l(P,K) = P_{11} + P_{12}K(I - P_{22}K)^{-1}P_{21} \tag{2}$$

Roughly speaking, the H-infinity norm is the peak magnitude of the transfer function in the frequency domain. It can also be given a time domain interpretation as the maximum (worst-case) 2-norm of the output over all normalised energy signal inputs in any direction. The difficulty with this formulation is that the uncertainties enter in the performance measure. It is possible to consider $\|N\|_\infty$ via the small gain theorem, which is the route pursued by standard H-infinity synthesis algorithms, but this leads to conservative controllers. This is because the small gain theorem assumes only that the uncertainties are stable and that their sizes are bounded, otherwise allowing them to be unstructured. The uncertainties, however, often possess some structure, and this is certainly the case in the systems under consideration within the present work. In order to deal with structured uncertainties[1], analysis and synthesis tools have been developed that rely on the structured singular value, a generalization of the familiar singular value.

This paper proceeds along the standard H-infinity route, reserving the application of the structured singular value approach for future publications.

[1] It can be shown that all robust performance problems possess a minimum structure [6].

B. Synthesis framework for 2-1 cascaded architecture

The aim of this section is to cast the cascaded sigma-delta architecture into the general control configuration of figure 1. This will be illustrated for the familiar 2-1 cascaded structure; future work will generalize the scheme.

The procedure involves two principal steps. The first step is to separate the signal paths into those that are affected by the uncertainties and those that are known (from simulation [3]) to be relatively insensitive to the uncertainties. Also, the uncertainties are "pulled out" and represented by (lower) LFTs. The second step then follows easily: the quantities and interconnections from figure 1 are identified for the system under consideration.

Figure 2 is the system interconnection diagram resulting from the first step. The main highlights are as follows. The standard additive white noise assumption is invoked to permit linear analysis (with the caveats as explained in [8]). It is the input to the quantizer of the first stage that is sensitive to imperfections in the noise filters, and hence, a separate signal path is required for the uncertain elements. The uncertainties are modelled as errors in the gains and poles of the noise shaping filters. The LFT representation for the gain error is easily derived (with $a = 0$ nominally):

$$\begin{aligned} 1 - a &= 0.5(1+\delta_a) \qquad \text{where } -1 \le \delta_a \le 1 \\ &= 0.5 + 0.5\,\delta_a (1-0)^{-1}.1 \\ &= F_l(M_a, \delta_a) \qquad \text{where } M_a = \begin{bmatrix} 0.5 & 0.5 \\ 1 & 1 \end{bmatrix} \end{aligned} \tag{3}$$

The pole uncertainty requires a little more work. The LFT representation proceeds from manipulating the input/output relationship for a first order noise filter with a real pole at $z = 1 - b$:

$$y = u + (1-b)\,z^{-1}\,y \tag{4}$$

The uncertainty in the pole ($= 1 - b$, $b = 0$ nominally) then has the same LFT form as that of the gain:

$$\begin{aligned} 1 - b &= 0.5(1+\delta_b) \qquad \text{where } -1 \le \delta_b \le 1 \\ &= F_l(M_b, \delta_b) \qquad \text{where } M_b = \begin{bmatrix} 0.5 & 0.5 \\ 1 & 1 \end{bmatrix} \end{aligned} \tag{5}$$

Finally, mapping the system variables to the ones in the general configuration in figure 1, one obtains:

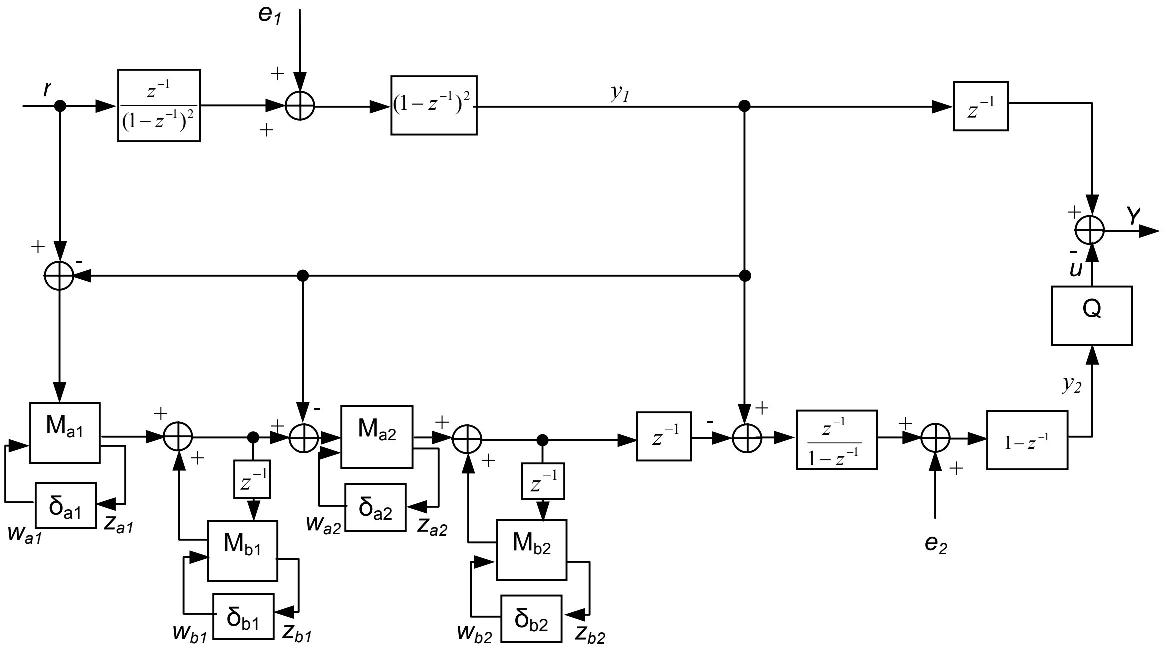

Fig. 2. System interconnection diagram for 2-1 cascaded sigma-delta converter.

$$w = \begin{bmatrix} e_1 \\ e_2 \\ r \end{bmatrix},\ v = y_2\ ,\ z = Y$$

$$w_\Delta = \begin{bmatrix} w_{a1} \\ w_{a2} \\ w_{b1} \\ w_{b2} \end{bmatrix},\ z_\Delta = \begin{bmatrix} z_{a1} \\ z_{a2} \\ z_{b1} \\ z_{b2} \end{bmatrix},\ \mathbf{\Delta} = \begin{bmatrix} \delta_{a1} & 0 & 0 & 0 \\ 0 & \delta_{a2} & 0 & 0 \\ 0 & 0 & \delta_{b1} & 0 \\ 0 & 0 & 0 & \delta_{b2} \end{bmatrix} \quad (6)$$

With the above set-up, the H-infinity analysis and design for robust performance requires one to consider $\|N\|_\infty$, where N is the closed-loop transfer function between the input vector $\begin{bmatrix} w \\ w_\Delta \end{bmatrix}$ and the output vector $\begin{bmatrix} z_\Delta \\ Y \end{bmatrix}$.

III. NUMERICAL RESULTS

The purpose of this section is to validate the robust synthesis framework developed in the previous section. The validity will be established in two ways. The first method examines the response of the system to a uniform noise input only in e_1 for the nominal controller and a set of uncertainties in the relevant parameters. For the nominal controller and parameter values, the response should be identically zero. The second method involves synthesizing the digital correction filter using the H-infinity approach for the zero perturbed system. It is easily shown that the nominal filter should have the form $(1-z^{-1})^2$ [3].

The system is installed in MATLAB using the **`sysic`** (system interconnection) command in the μ-Analysis and Synthesis Toolbox [5]. The results of simulating various perturbed systems subject to a uniform random noise input on e_1 is given in figure 3. It is clear from figure 3 that the response is indeed zero for the zero perturbation case. A discrete H-infinity controller is synthesized for the nominally perturbed system using the command **`dhfsyn`**. The synthesized controller turns out precisely to have the required form.

IV. CONCLUSIONS

This paper has presented a new framework in which robust digital correction filters may be designed to tackle the mismatch problem associated with cascaded sigma-delta modulators. The approach was illustrated for the 2-1 architecture and validated by reproducing established results.

Future work will mainly concentrate on extending the approach to encompass more general cascaded structures and using this to tackle the synthesis problem directly. In the first instance worst-case H-infinity design will be explored which would lead to conservative filters. The more difficult task of developing filters satisfying stricter robust performance

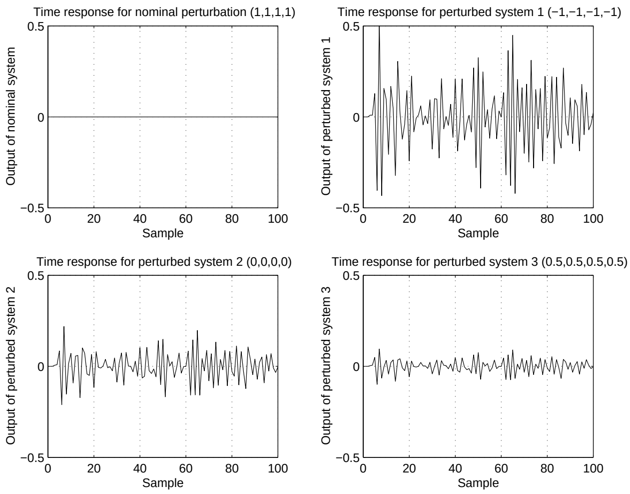

Fig. 3: Time responses of Matlab model of 2-1 architecture to uniform random noise input on e_1 perturbations are (δ_{a1}, δ_{a2}, δ_{b1}, δ_{b2}).

criteria will require the use of structured singular value tools.

The ideas presented in this paper are not limited to cascaded structures – other modulator topologies which contain more than one quantizer in the signal path (such as parallel architectures [9,10]), and therefore suffer from mismatch effects, are likely to benefit from a robust correction scheme.

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